

# WORKSHOP CALCULATION & SCIENCE

(NSQF)

2<sup>nd</sup> YEAR

(As per Revised Syllabus July 2022)

**SURVEYOR**



Directorate General of Training

DIRECTORATE GENERAL OF TRAINING  
MINISTRY OF SKILL DEVELOPMENT & ENTREPRENEURSHIP  
GOVERNMENT OF INDIA



**NATIONAL INSTRUCTIONAL  
MEDIA INSTITUTE, CHENNAI**

Post Box No. 3142, CTI Campus, Guindy, Chennai - 600 032

**Workshop Calculation & Science**  
**Surveyor - 2<sup>nd</sup> Year NSQF**  
**As per Revised Syllabus July 2022**

**Developed & Published by**



**National Instructional Media Institute**

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## FOREWORD

The Government of India has set an ambitious target of imparting skills one out of every four Indians, to help them secure jobs as part of the National Skills Development Policy. Industrial Training Institutes (ITIs) play a vital role in this process especially in terms of providing skilled manpower. Keeping this in mind, and for providing the current industry relevant skill training to Trainees, ITI syllabus has been recently updated with the help of comprising various stakeholder's viz. Industries, Entrepreneurs, Academicians and representatives from ITIs.

The National Instructional Media Institute (NIMI), Chennai, has now come up with instructional material to suit the revised curriculum for **Workshop Calculation & Science - Surveyor 2<sup>nd</sup> Year** NSQF (Revised 2022) under CTS will help the trainees to get an international equivalency standard where their skill proficiency and competency will be duly recognized across the globe and this will also increase the scope of recognition of prior learning. NSQF trainees will also get the opportunities to promote life long learning and skill development. I have no doubt that with NSQF the trainers and trainees of ITIs, and all stakeholders will derive maximum benefits from these IMPs and that NIMI's effort will go a long way in improving the quality of Vocational training in the country.

The Director General of Training, Executive Director & Staff of NIMI and members of Media Development Committee deserve appreciation for their contribution in bringing out this publication.

Jai Hind

**ATUL KUMAR TIWARI, I.A.S.**

Secretary  
Ministry of Skill Development & Entrepreneurship,  
Government of India.

December 2023  
New Delhi - 110 001

## PREFACE

The National Instructional Media Institute(NIMI) was set up at Chennai, by the Directorate General of Training, Ministry of skill Development and Entrepreneurship, Government of India, with the technical assistance from the Govt of the Federal Republic of Germany with the prime objective of developing and disseminating instructional Material for various trades as per prescribed syllabus and Craftsman Training Programme(CTS) under NSQF levels.

The Instructional materials are developed and produced in the form of Instructional Media Packages (IMPs), consisting of Trade Theory, Trade Practical, Test and Assignment Book, Instructor Guide and Wall charts. The above material will enable to achieve overall improvement in the standard of training in ITIs.

A national multi-skill programme called SKILL INDIA, was launched by the Government of India, through a Gazette Notification from the Ministry of Finance (Dept of Economic Affairs), Govt of India, dated 27th December 2013, with a view to create opportunities, space and scope for the development of talents of Indian Youth, and to develop those sectors under Skill Development.

The emphasis is to skill the Youth in such a manner to enable them to get employment and also improve Entrepreneurship by providing training, support and guidance for all occupation that were of traditional types. The training programme would be in the lines of International level, so that youths of our Country can get employed within the Country or Overseas employment. The **National Skill Qualification Framework (NSQF)**, anchored at the National Skill Development Agency(NSDA), is a Nationally Integrated Education and competency-based framework, to organize all qualifications according to a series of **levels of Knowledge, Skill and Aptitude**. Under NSQF the learner can acquire the Certification for Competency needed at any level through formal, non-formal or informal learning.

The **Workshop Calculation & Science - Surveyor 2<sup>nd</sup> Year NSQF (Revised 2022)** under CTS is one of the book developed by the core group members as per the NSQF syllabus.

The **Workshop Calculation & Science - Surveyor 2<sup>nd</sup> Year NSQF (Revised 2022)** under CTS as per NSQF is the outcome of the collective efforts of experts from Field Institutes of DGT, Champion ITI's for each of the Sectors, and also Media Development Committee (**MDC**) members and Staff of **NIMI**. NIMI wishes that the above material will fulfill to satisfy the long needs of the trainees and instructors and shall help the trainees for their Employability in Vocational Training.

NIMI would like to take this opportunity to convey sincere thanks to all the Members and Media Development Committee (MDC) members.

Chennai - 600 032

EXECUTIVE DIRECTOR



## ACKNOWLEDGEMENT

The National Instructional Media Institute (NIMI) sincerely acknowledge with thanks the co-operation and contribution of the following Media Developers to bring this IMP for **Workshop Calculation & Science - Surveyor 2<sup>nd</sup> Year** as per NSQF Revised 2022.

### MEDIA DEVELOPMENT COMMITTEE MEMBERS

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### NIMI CO-ORDINATORS

Shri. Nirmalya Nath	-	Deputy General Manager, NIMI, Chennai - 32.
Shri. G. Michael Johny	-	Manager, NIMI, Chennai - 32.

NIMI records its appreciation of the **Data Entry, CAD, DTP Operators** for their excellent and devoted services in the process of development of this IMP.

NIMI also acknowledges with thanks, the efforts rendered by all other staff who have contributed for the development of this book.

## INTRODUCTION

The material has been divided into independent learning units, each consisting of a summary of the topic and an assignment part. The summary explains in a clear and easily understandable fashion the essence of the mathematical and scientific principles. This must not be treated as a replacement for the instructor's explanatory information to be imparted to the trainees in the classroom, which certainly will be more elaborate. The book should enable the trainees in grasping the essentials from the elaboration made by the instructor and will help them to solve independently the assignments of the respective chapters. It will also help them to solve the various problems, they may come across on the shop floor while doing their practical exercises.

The assignments are presented through 'Graphics' to ensure communications amongst the trainees. It also assists the trainees to determine the right approach to solve the problems. The required relevant data to solve the problems are provided adjacent to the graphics either by means of symbols or by means of words. The description of the symbols indicated in the problems has its reference in the relevant summaries.

At the end of the exercise wherever necessary assignments, problems are included for further practice.

### Time allotment - 2<sup>nd</sup> Year : 40 Hrs

Time allotment for each title of exercises has been given below. **Workshop Calculation & Science - Surveyor** 2<sup>nd</sup> Year NSQF Revised Syllabus 2022.

S.No	Title	Exercise No.	Time in Hrs
1	Area of cut out regular surfaces and area of irregular surfaces	2.1.01 - 2.1.03	14
2	Algebra	2.2.04 & 2.2.05	12
3	Profit and Loss	2.3.06	4
4	Estimation and Costing	2.4.07 - 2.4.14	10
		<b>Total</b>	<b>40 Hrs</b>

## LEARNING / ASSESSABLE OUTCOME

On completion of this book you shall be able to

- **Demonstrate basic mathematical concept and principles to perform practical operations.**
- **Understand and explain basic science in the field of study.**

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# SYLLABUS

2<sup>nd</sup> Year

## Workshop Calculation & Science - Surveyor Revised syllabus July 2022 under CTS

S.no.	Syllabus	Time in Hrs
I	<b>Area of cut-out regular surfaces and area of irregular surfaces</b> 1 Area of cut-out regular surfaces – circle, segment and sector of circle 2 Related problems of area of cut-out regular surfaces – circle, segment and sector of circle 3 Area of irregular surfaces and application related to shop problems	14
II	<b>Algebra</b> 1 Addition, Subtraction, Multiplication & Divisions 2 Algebra – Theory of indices, Algebraic formula, related problems	12
III	<b>Profit and Loss</b> 1 Simple problems on profit & loss 2 Simple and compound interest	4
IV	<b>Estimation and Costing</b> 1 Simple estimation of the requirement of material etc., as applicable to the trade 2 Problems on estimation and costing	10
	<b>Total</b>	<b>40</b>

Area of cut-out regular surfaces - Circle, segment and sector of circle

**Circle (Fig 1)**

It is the path of a point which is always equal from its centre is called a circle.

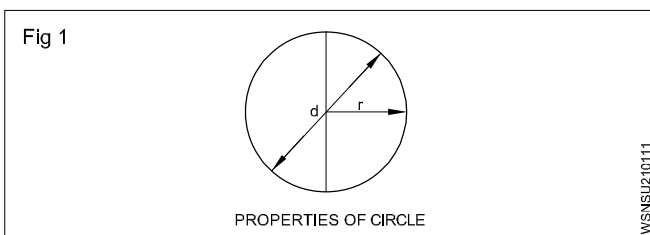
$r$  = radius of the circle

$d$  = diameter of the circle

Area of the circle =  $\pi r^2$

$$(or) = \frac{\pi}{4} d^2 \text{ unit}^2$$

Circumference of the circle =  $2\pi r$  (or)  $\pi d$  unit



**Sector of a circle (Fig 2)**

The area bounded by an arc and two side radius is called the sector of a circle. In the figure given ABC is the sector of a circle.

$r$  = radius of the circle

$\theta$  = Angle of sector in degrees

Area of sector ABC

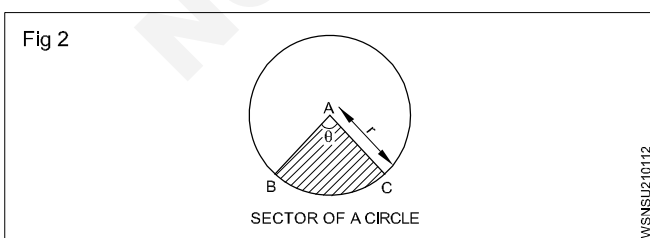
$$= \frac{\pi r^2 \times \theta}{360^\circ} \text{ unit}^2$$

$$\text{Area of sector} = \frac{\text{Length of arc of sector} \times \text{radius}}{2} \text{ unit}^2$$

$$\text{Length of the arc } l = 2\pi r \times \frac{\theta}{360^\circ} \text{ unit}$$

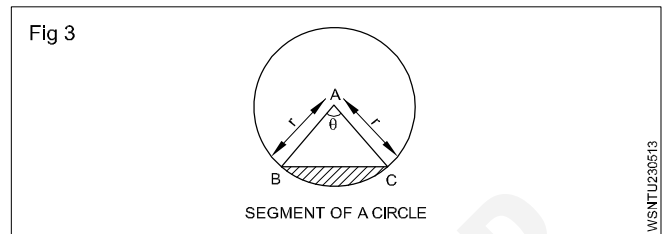
Perimeter of the sector =  $l + 2r$  unit

$r$  = radius



**Segment of a circle (Fig 3)**

When a circle is divided into two by drawing a line, the bigger part is called segment of the circle and the smaller part is also called segment of the circle.



Area of the smaller segment

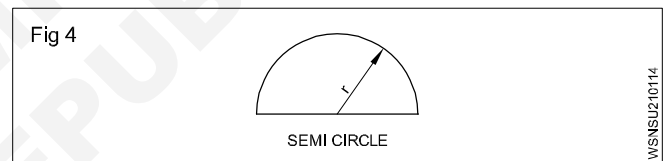
$$= \text{Area of the sector} - \text{Area of } \Delta ABC$$

Area of the greater segment

$$= \text{Area of the circle} - \text{Area of smaller segment}$$

**Semi Circle (Fig 4)**

- A semi circle is a sector whose central angle is  $180^\circ$ .



- Length of arc of semi circle

$$l = 2\pi r \times \frac{180^\circ}{360^\circ} = 2\pi r \times \frac{1}{2} \text{ unit}$$

$$= \pi r \text{ unit}$$

$$\text{Area of semi circle} = \frac{\pi r^2}{2} \text{ unit}^2$$

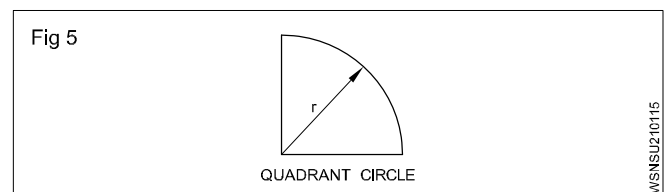
$$\text{Perimeter of a semi circle} = \frac{2\pi r}{2} + 2r$$

$$= \pi r + 2r$$

$$= r(\pi + 2) \text{ unit}$$

**Quadrant of a circle (Fig 5)**

- A quadrant of a circle is a sector whose central angle is  $90^\circ$ .



- Length of arc of quadrant of a circle

$$\begin{aligned} l &= 2\pi r \times \frac{90^\circ}{360^\circ} \\ &= 2\pi r \times \frac{1}{4} \\ &= \frac{\pi r}{2} \end{aligned}$$

$$\text{Area of quadrant of a circle} = \frac{\pi r^2}{4} \text{ unit}^2$$

$$\text{Perimeter of a quadrant} = \frac{2\pi r}{4} + 2r$$

$$\begin{aligned} &= \frac{\pi r}{2} + 2r \\ &= r \left( \frac{\pi}{2} + 2 \right) \text{ unit} \end{aligned}$$

#### Examples :

- 1 Find the area of a sector of a circle whose radius is 14 cm and the length of the arc of the sector is 28 cm.

Radius of sector  $r = 14$  cm

Length of arc of sector = 28 cm

$$\text{Length of arc of sector } (l) = \frac{\theta}{360^\circ} \times 2\pi r \text{ unit}$$

$$28 = \frac{\theta}{360^\circ} \times 2 \times \frac{22}{7} \times 14 \text{ unit}$$

$$\theta = \frac{28 \times 360^\circ \times 7}{2 \times 22 \times 14} = 114.55^\circ$$

$\therefore$  Angle of sector  $\theta = 114.55^\circ$

$$\begin{aligned} \therefore \text{Area of sector} &= \frac{\theta}{360^\circ} \times \pi r^2 \text{ unit}^2 \\ &= \frac{114.55}{360^\circ} \times \frac{22}{7} \times 14 \times 14 \text{ cm}^2 \\ &= 196 \text{ cm}^2 \end{aligned}$$

**Area of sector = 196 cm<sup>2</sup>**

- 2 If the circumference of a circle is 44 cm, find its area. (Take  $\pi = \frac{22}{7}$ )

#### Solution

$\therefore$  Let (d) = diameter of circle

$\therefore$  Circumference of circle =  $\pi d$

$$\therefore 44 = \pi \cdot d$$

$$d = \frac{44}{\pi} = 44 \div \pi$$

$$= 44 \div \frac{22}{7}$$

$$= 44 \times \frac{7}{22}$$

$$= 14 \text{ cm}$$

$\therefore$  Diameter of circle (d) = 14 cm

$$\therefore \text{Area of circle} = \frac{\pi}{4} d^2 \text{ unit}^2$$

$$= \pi \times \frac{1}{4} d^2$$

$$= \frac{22}{7} \times \frac{1}{4} \times 14 \times 14$$

$$= 154 \text{ cm}^2$$

Area of circle = 154 cm<sup>2</sup>

- 3 Find the remaining area of circle of 10 cm dia after inscribing triangle of 5 cm base and 10 cm height.

#### Solution

$$\begin{aligned} \text{(i) Area of the circle} &= \frac{\pi}{4} d^2 \\ &= \frac{22 \times 10 \times 10}{7 \times 4} \text{ Unit}^2 \\ &= \frac{550}{7} = 78.57 \text{ cm}^2 \end{aligned}$$

(ii) Area of the triangle inscribed in this circle

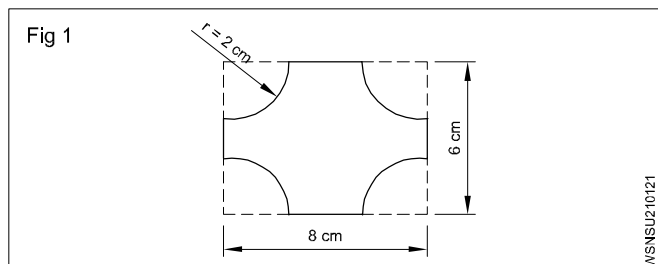
$$= \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{10 \times 5}{2} = 25 \text{ sq.cm}$$

Remaining area = 78.57 - 25

Remaining area of circle = 53.57 cm<sup>2</sup>

- 4 A rectangular sheet of metal measures 8 cm and 6 cm. Four quadrants of circles each of radius 2 cm are cut away at corners. Find the area of the remaining portion.



$$\begin{aligned} \text{Area of rectangular sheet} &= 8 \times 6 \\ &= 48 \text{ cm}^2 \end{aligned}$$

There are four quadrants of a circle, each of radius 2 cm cut away at the corners. Quadrant of circle means 1/4th of circle.

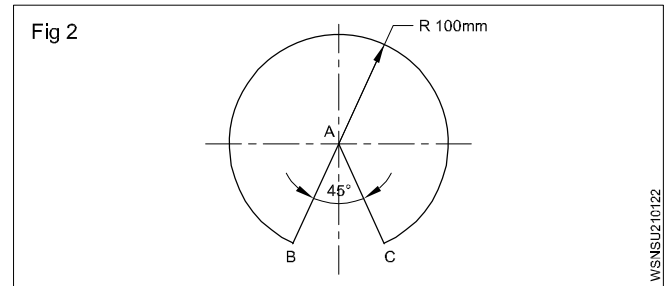
$$4 \text{ quadrant of circles} = 4 \times \frac{1}{4} \text{ of circle} = 1 \text{ circle}$$

$$\begin{aligned} \text{Area of 4 quadrant circles} &= \text{Area of one circle} \\ &= \pi r^2 \\ &= \frac{22}{7} \times 2 \times 2 \\ &= 12.57 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of remaining portion} &= \text{Area of rectangular sheet} - \\ &\quad \text{Area of four quadrant circles} \\ &\quad \text{cut at corners.} \\ &= 48 - 12.57 \\ &= 35.428 \text{ cm}^2 \\ &= \text{say } 35.43 \text{ cm}^2 \end{aligned}$$

$$\text{Area of remaining portion} = 35.43 \text{ cm}^2$$

## 5 Find the perimeter of the given circular disc.



### Sector :

$$\begin{aligned} r &= 100 \text{ mm} \\ \theta &= 360^\circ - 45^\circ = 315^\circ \end{aligned}$$

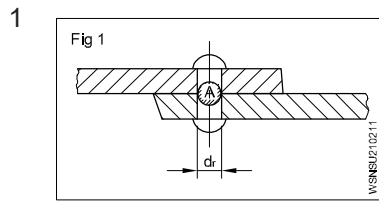
$$\begin{aligned} \ell &= \frac{\theta}{360} \times 2\pi r \text{ unit} \\ &= \frac{315}{360} \times 2 \times \pi \times 100 \text{ mm} \end{aligned}$$

$$\ell = 550 \text{ mm}$$

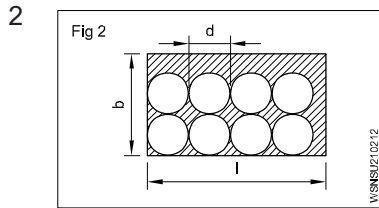
$$\begin{aligned} \text{Perimeter of the given circular Disc} &= \ell + 2r \\ &= 550 + 200 = 750 \text{ mm} \end{aligned}$$

$$\text{Perimeter of the given circular Disc} = 750 \text{ mm}$$

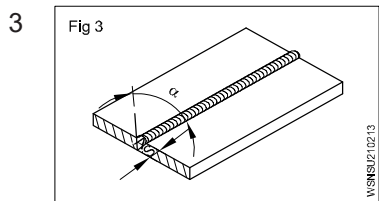
Related problems of area of cut out regular surfaces - circle, segment and sector of circle



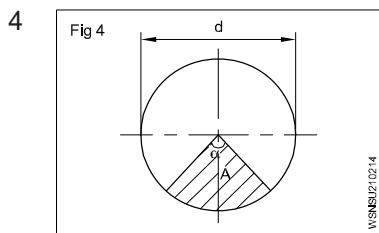
$d_t = 21 \text{ mm}$   
 $A_t = \text{_____ mm}^2$



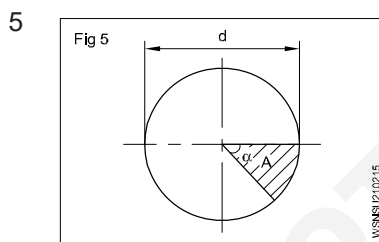
$l = 750 \text{ mm}$   
 $b = 400 \text{ mm}$   
 $d = 180 \text{ mm}$   
 Area of sheet = \_\_\_\_\_



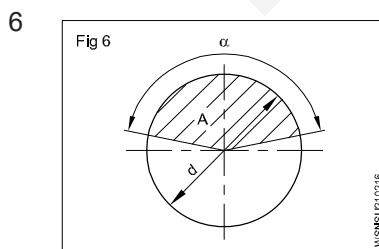
$\alpha = 60^\circ$   
 $r = 9.2 \text{ mm}$   
 $A$  of sector = \_\_\_\_\_  $\text{mm}^2$



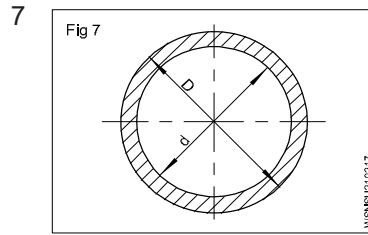
$A = \text{Area of sector} = 140 \text{ mm}^2$   
 $d$  of the circle = 30 mm  
 $\alpha = \text{_____}^\circ$



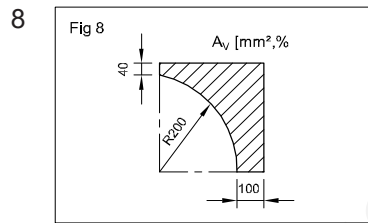
$d = 380 \text{ mm}$   
 No. of sectors of equal area = 8  
 $A$  of each sector = \_\_\_\_\_  $\text{mm}^2$   
 $\alpha = \text{_____}^\circ$   
 length of arc of each sector = \_\_\_\_\_ mm



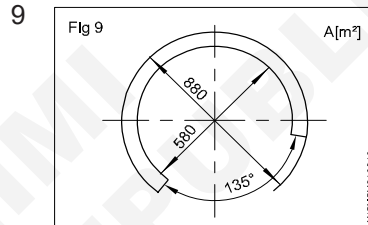
$\alpha = 160^\circ$   
 $A = 0.893 \text{ m}^2$   
 $d = \text{_____ mm}$



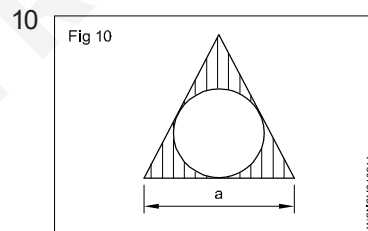
$D = 38 \text{ mm}$   
 $d = 32 \text{ mm}$   
 Cross sectional area = \_\_\_\_\_  $\text{mm}^2$



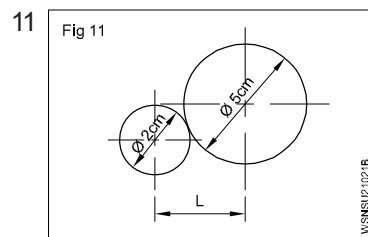
$A_v$  (Area of shaded part) = \_\_\_\_\_  $\text{mm}^2$   
 $A_v = \text{\% of (Area of rectangle)} A_1$



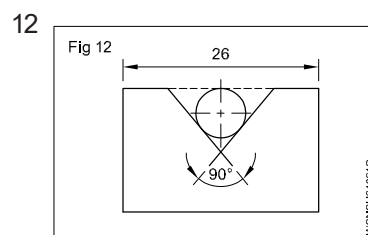
$D = 880 \text{ mm}$   
 $d = 580 \text{ mm}$   
 Angle of cut off sector =  $135^\circ$   
 Area of the remaining portion,  $A = \text{_____ mm}^2$



Equilateral triangle of side  $a = 6 \text{ cm}$   
 Radius of circle = 1.732 cm  
 Shaded area = \_\_\_\_\_



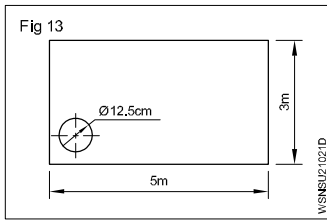
Two plugs having diameters 2 cm and 5 cm are placed on a surface plate touching each other. calculate the distance 'L' in the figure.



90° vee block is 26 mm wide at the top of the vee block. What dia. of shaft when laid in the vee block will have its top surface just level with the top of the vee block.

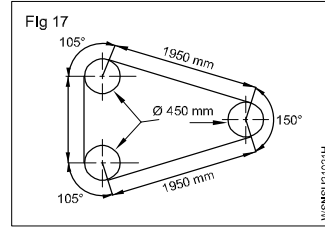


13



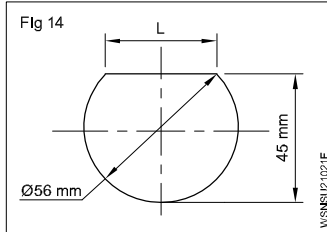
From a sheet of 5m × 3m how many circular pieces of 12.5 cm dia can be cut.

17



The arrangement of a band saw blade is shown in the figure given below. Find out the length of the saw blade.

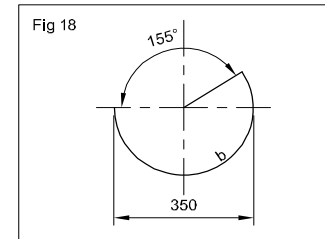
14



Find out 'L' from the given sketch.

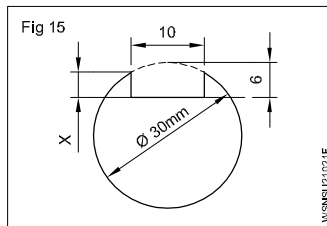
18 Calculate the area covered by 3 equal circles of radius 2.8 cm touches one another.

19



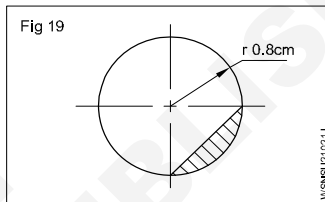
$\alpha = 155^\circ$   
 $d = 350 \text{ mm}$   
 $b = \text{---} \text{ mm}$

15



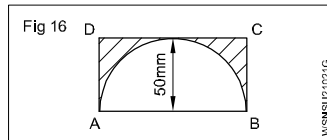
Find the value of 'x' in the fig.

20



Find the area of shaded portion.

16

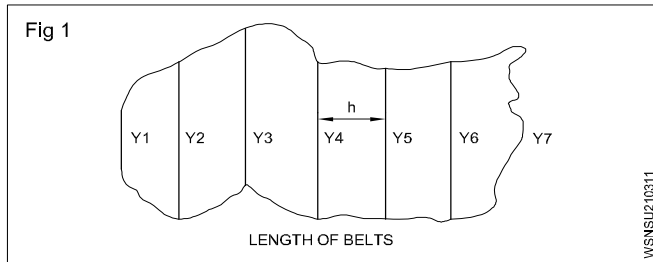


Area of the shaded portion = \_\_\_\_\_ mm<sup>2</sup>.

Area of irregular surfaces and application related to shop problems

Area of irregular surface

Surface area of irregular figures can be obtained by applying either, Simpson's rule or trapezoidal rule. Area found by Simpson's rule is more accurate than trapezoidal rule. However accurate area can be obtained if the number of ordinates are more i.e interval between ordinates is so small as possible. (Fig 1)



i Area as per Simpson's rule

$$\text{Area} = \frac{h}{3} [(y_1 + y_7) + 4(y_2 + y_4 + y_6) + 2(y_3 + y_5)]$$

where

h = interval between ordinates

ii Area as per trapezoidal rule

$$\text{Area} = \frac{h}{2} [(y_1 + y_7) + 2(y_2 + y_3 + y_4 + y_5 + y_6)]$$

where

h = interval between ordinates

Calculate the area enclosed between the chain line, the edge and the end offsets by

The offsets were taken from a chain line to a edge.

Distance (M)	0	5	10	15	20	25	30	35
Off set (M)	4	3	2	5	1	2	3	5

(a) Simpson's rule

(a) Simpson's rule

$$A = \frac{h}{3} [(y_1 + y_8) + 4(y_2 + y_4 + y_6) + 2(y_3 + y_5 + Y_7)] \text{ unit}^2$$

$$A = \frac{5}{3} [(4 + 5) + 4(3 + 5 + 2) + 2(2 + 1 + 3)] \text{ m}^2$$

$$= \frac{5}{3} [9 + 4(10) + 2(6)]$$

$$= \frac{5}{3} [9 + 40 + 12]$$

$$= \frac{5}{3} \times 61 = 101.66$$

$$= 101.7 \text{ m}^2$$

(b) Trapezoidal rule

$$A = \frac{h}{2} [(y_1 + y_8) + 2(y_2 + y_3 + y_4 + y_5 + y_6 + Y_7)] \text{ unit}^2$$

$$A = \frac{5}{2} [(4 + 5) + 2(3 + 2 + 5 + 1 + 2 + 3)] \text{ m}^2$$

$$= \frac{5}{2} [9 + 2(16)] \text{ m}^2$$

$$= \frac{5}{2} [9 + 32] \text{ m}^2$$

$$A = \frac{5}{2} \times 41 \text{ m}^2$$

$$= 102.5 \text{ m}^2$$

Calculation of the area of an irregular surface

In this Calculation the area of an irregular surface may be determined as follows.

In this method of calculation a chain line known as base line to be laid through the centre of the area of the surface.

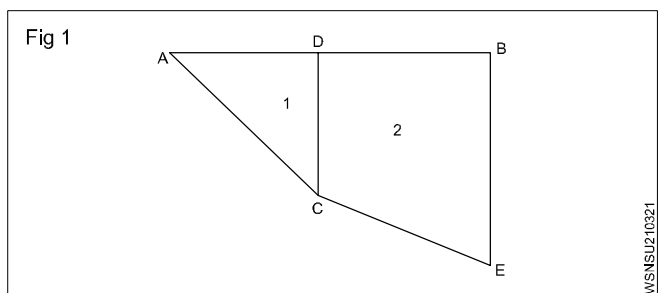
The offset are taken to the boundary points in the order of their chainages on both the sides of the base line.

The chain line and offsets are noted down.

With reference to the notes the boundary points are plotted and the area to be divided into number of triangles and trapezium according to the shape.

Example

Now apply the geometrical formulae for calculation according to the shape of the figure. (Fig 1)



Chainline = AB

Offsets = C,E

1 Area of triangle

$$\frac{1}{2} \times \text{base} \times \text{height}$$

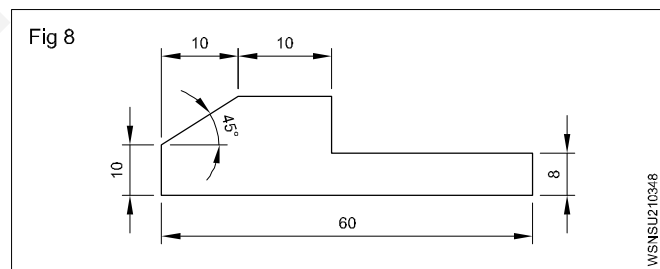
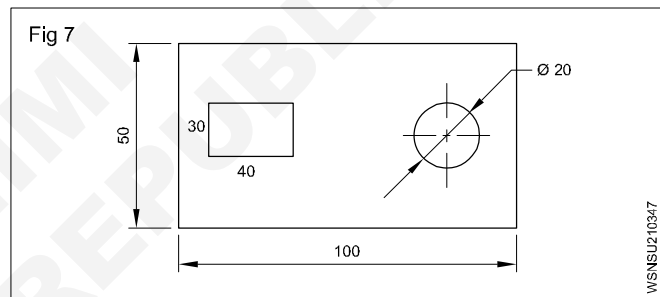
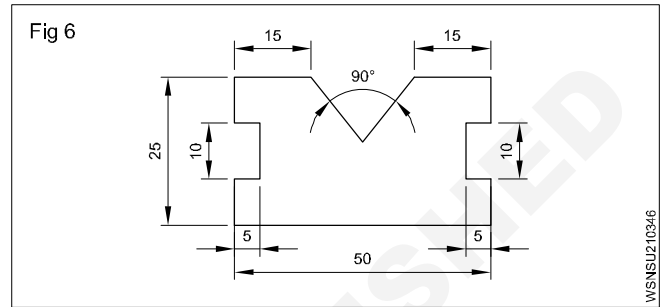
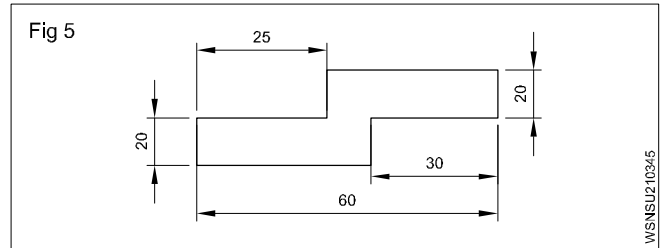
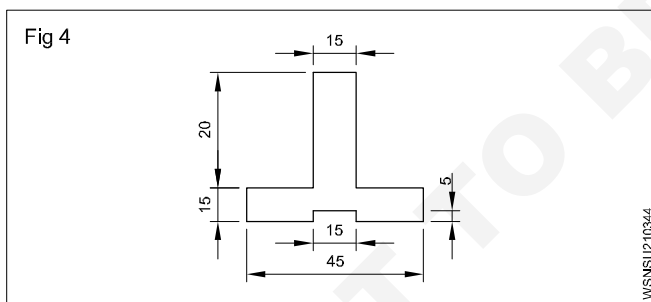
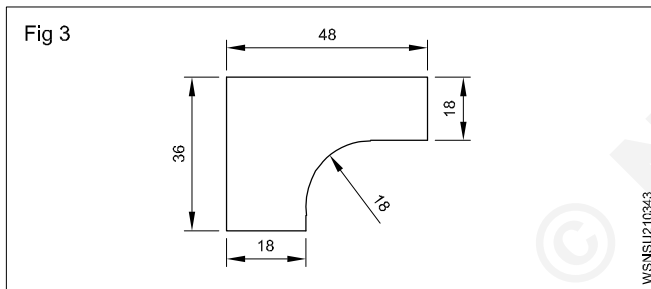
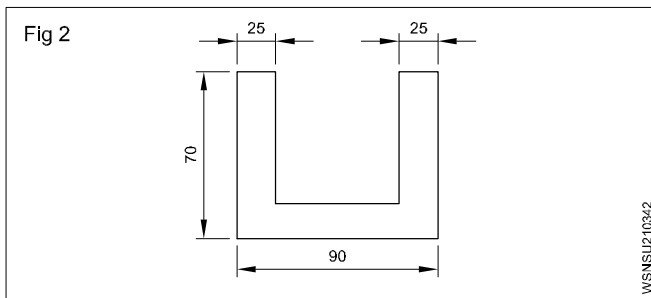
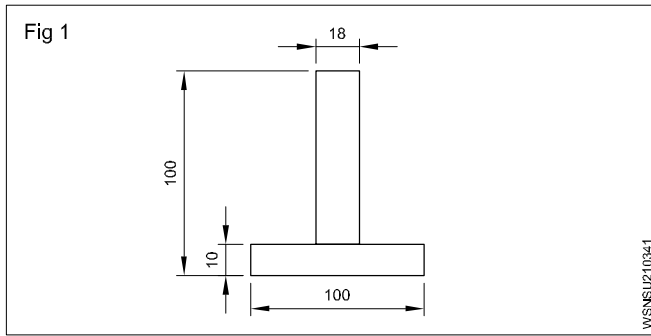
2 Area of trapezium

$$\frac{\text{base} (a + b)}{2} \times \text{height}$$



# Assignment

Calculate the area of the irregular surfaces given below.



Note : All dimension are in mm.

## Algebra - Addition, subtraction, multiplication & division

### Introduction

Algebra is a form of mathematics in which letters may be used in place of unknown. In this mathematics numbers are also used in addition to the letters and the value of number depends upon its place. For example in  $3x$  and  $x^3$ , the place of  $x$  is different. In  $3x = 3$  is multiplied with  $x$ , whereas in  $x^3 - 3$  is an Index of  $x$ .

### Positive and negative numbers

Positive numbers have a + sign in front of them, and negative numbers have – sign in front of them. The same applies to letters also.

**Example**  $+x$ ,  $-y$ .

+8 or simply 8 positive number.

–8 negative number.

### Addition and subtraction

Two positive numbers are added, by adding their absolute magnitude and prefix the plus sign.

To add two negative numbers, add their absolute magnitude and prefix the minus sign.

To add a positive and a negative number, obtain the difference of their absolute magnitudes and prefix the sign of the number having the greater magnitude.

$$+7 + 22 = +29$$

$$(-8) - 34 = -42$$

$$(-27) + 19 = -8$$

$$44 + (-18) = +26$$

$$37 + (-52) = -15$$

### Multiplication of positive and negative numbers

The product of two numbers having like signs is positive and the product of two numbers with unlike signs is negative. Note that, where both the numbers are negative, their product is positive.

**Ex.**  $-20 \times -3 = 60$

$$5 \times 8 = 40$$

$$4 \times -13 = -52$$

$$-5 \times 12 = -60$$

### Division

The number that is divided is the dividend, the number by which we are dividing is the divisor and the answer is the quotient. If the signs of the dividend and the divisor are the same then the quotient will have a + sign. If they are unlike then the quotient will have a negative sign.

$$\frac{+28}{+4} = +7$$

$$\frac{+56}{-4} = -14$$

$$\frac{-72}{+9} = -8$$

$$\frac{-96}{-6} = +16$$

**When an expression contains addition, subtraction, multiplication and division, perform the multiplication and division operations first and then do the addition and subtraction.**

### Example

$$12 \times 8 - 6 + 4 \times 12 = 96 - 6 + 48 = 138$$

$$102 \div 6 - 6 \times 2 + 3 = 17 - 12 + 3 = 8$$

### Parentheses and grouping symbols

( ) Brackets

{ } Braces

$$7 + (6-2) = 7 + 4 = 11$$

$$6 \times (8-5) = 6 \times 3 = 18$$

### Parentheses

These are symbols that indicate that certain addition and subtraction operations should precede multiplication and division. They indicate that the operations within them should be carried out completely before the remaining operations are performed. After completing the grouping, the symbols may be removed.

In an expression where grouping symbols immediately preceded or followed by a number but with the signs of operation omitted, it is understood, that multiplication should be performed.

Grouping symbols are used when subtraction and multiplication of negative number is done.

To remove grouping symbols which are preceded by negative signs, the signs of all terms inside the grouping symbols must be changed (from plus to minus and minus to plus).

Parentheses which are preceded by a plus sign may be removed without changing the signs of the terms within the parentheses.

When one set of grouping symbols is included within another set, remove the innermost set first.

When several terms connected by + or – signs contain a common quantity, this common quantity may be placed in front of a parentheses.

$$8 + 6(4-1) = 8 + 6 \times 3 = 26$$

$$(6+2)(9-5) = 8 \times 4 = 32$$

Plus 4 less negative 7 is written as  $4 - (-7)$ .

Plus 4 times negative 7 is written as  $4(-7)$ .

$$4 - (-7) = 4 + 7 = 11$$

$$8 - (7-4) = 8 - 3 = 5$$

$$3 + (-8) = 3 - 8 = -5$$

$$7 + (4 - 19) = 7 + (-15) = 7 - 15 = -8$$

$$\begin{aligned} 3 \{40 + (7 + 5) (8-2)\} \\ = 3 \{40 + 12 \times 6\} \\ = 3 \times 112 = 336. \end{aligned}$$

$8x + 12$  - quantity 4 may be factored out giving the expression  $8x + 12$  as  $4(2x + 3)$ .

The innermost set in a grouping symbols of an expression is to be simplified first.

### Algebraic symbols and simple equations

#### Algebraic symbol

An unknown numerical value of a quantity is represented by a letter which is the algebraic symbol.

#### Factor

A factor is any one of the numbers or letters or groups which when multiplied together give the expression. Factors of 12 are 4 and 3 or 6 and 2 or 12 and 1.

$8x + 12$  is the expression and this may be written as  $4(2x + 3)$ , 4 and  $(2x + 3)$  are the factors.

#### Algebraic terms

If an expression contains two or more parts separated by either + or -, each part is known as the term.

$y - 5x$  is the expression.  $y$  and  $-5x$  are the terms.

The sign must precede the term.

#### Kinds of terms:

##### 1 Like terms

a  $13a, 15a, 19a, -12a, -18a$

b  $5xy, 11xy, -xy, -14xy$

c  $27m^2, 25m^2, -3m^2, 11m^2$

##### 2 Unlike terms

a  $3ac, -4b, 8x, 3yz$

b  $2xy, y^2, a^2b, xz, 3bc$

c  $13m^2n, 3mn^2, 14lm^2, 15a^2b, 5lm$

#### Examples :

$$\begin{aligned} 1 \text{ Add } 7a, -2a, a, 3a \\ 7a + (-2a) + (a) + 3a \\ 7a - 2a + a + 3a \\ = 11a - 2a \\ = 9a \end{aligned}$$

$$\begin{aligned} 2 \text{ Add } 25xy, + 2xy, - 6xy, - 3xy \\ 25xy + 2xy + (-6xy) + (-3xy) \\ = 27xy - 9xy \\ = 18xy \end{aligned}$$

$$\begin{aligned} 3 \text{ Add } 9m, + 4m, - 2 \\ 9m + 4m + (-2) \\ 9m + 4m - 2 \\ = 13m - 2 \end{aligned}$$

#### Coefficient

When an expression is formed into factors whose product is the expression, then each factor is the coefficient of the remaining factors.

$$48x = 4 \times 12 \times x$$

4 is the coefficient of  $12x$ .  $x$  is the coefficient of 48.

#### Equation

It is a statement of equality between numbers or numbers and algebraic symbols.

$$12 = 6 \times 2, 13 + 5 = 18.$$

$$2x + 9 = 5, y - 7 = 4y + 5.$$

#### Simple equation

Equations involving algebraic symbols to the first power are simple equations.

$$2x + 4 = 10. \quad 4x + 12 = 14.$$

#### Addition

$$\begin{aligned} 1 \quad 8a + 12b - a - 14b \\ = 8a - a + 12b - 14b \\ = 7a - 2b \\ 2 \quad 14a + 3a + 25b + 2b + b \\ = 17a + 28b \\ 3 \quad (2a + 3b - c) + (4a - b - c) + (a - 8) \\ 2a + 3b - c + 0 \\ 4a - b - c + 0 \\ a + 0 + 0 - 8 \\ \hline 7a + 2b - 2c - 8 \end{aligned}$$

$$\begin{aligned} 4 \text{ Add : } (3x + 3z) ; (5x - 4y) ; (9y - 3z) \\ 3x + 0 + 3z \\ 5x - 4y + 0 \\ 0 + 9y - 3z \\ \hline 8x + 5y \end{aligned}$$

## Subtraction

1  $38xy - 15xy = 23xy$

2 Subtract  $3xy$  from  $-4xy$

$$\begin{array}{r} -4xy \\ +3xy \\ \hline (-) \\ \hline -7xy \\ \hline \end{array}$$

3 Subtract  $5x$  from  $12x$

$$= 12x - (5x)$$

$$= 12x - 5x$$

$$= 7x$$

4 Subtract  $18x$  from  $7x$

$$= 7x - (18x)$$

$$= 7x - 18x$$

$$= -11x$$

5 Subtract  $3x - 2y$  from  $4y - 2x$

$$= (4y - 2x) - (3x - 2y)$$

$$= 4y - 2x - 3x + 2y$$

$$= 6y - 5x$$

## Addition and subtraction

Quantities with algebraic symbols are added or subtracted by considering those terms involving same symbols and powers.

Example

$$\begin{aligned} 1. \quad & 10x + 14 - 7y^2 - 11a + 2x - 4 - 3y^2 - 4a + 8 \\ & = 10x + 2x - 7y^2 - 3y^2 - 11a - 4a + 14 - 4 + 8 \\ & = 12x - 10y^2 - 15a + 18 \end{aligned}$$

2.  $2x = 10$ ,  $2x + 6 = 10 + 6$

3.  $y + 12 = 20$ ,  $y + 12 - 8 = 20 - 8$

4.  $x + 10 = 12$ ,

$$x + 10 - 10 = 12 - 10$$

5.  $3x = 6$ ,  $2 \times 3x = 2 \times 6$ ,  $6x = 12$

6.  $5y = 20$ ,  $\frac{5y}{5} = \frac{20}{5}$ .

The same number may be added or subtracted to both members of an equation without changing its equality.

Each member of an equation may be multiplied or divided by the same number or symbol without changing its equality.

The equality of an equation is not altered when the numbers or symbols are added or subtracted from both sides. Multiplication and division by the same numbers or symbols on both sides also will not affect the equality.

## Transposition of the terms of the equations

= equals to

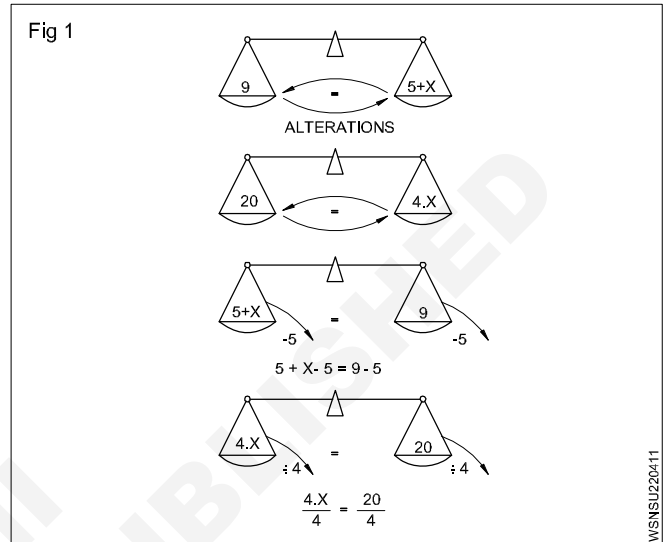
+ plus

- minus

x multiply

÷ divided by

## Concept of equality (Fig 1)



An equation can be compared to a pair of scales which always remain in equilibrium. The two sides of the equation can fully be transposed.  $9 = 5 + x$  may also be written as  $5 + x = 9$ .

We must always perform the same operation on both sides of the equation to keep the equilibrium. Add or subtract the same amount from both sides.  $5 + x = 9$  By adding 3 on both sides, the equation becomes  $5 + x + 3 = 9 + 3$  or  $x + 8 = 12$ .

$5 + x = 9$  Subtract 5 from both sides then  $5 + x - 5 = 9 - 5$ .  
 $x = 4$ .

5 is transposed from left side to the right side by changing its sign from + to -.

$\frac{x}{4} = 20$ . Multiply both sides by 4. Then  $\frac{x}{4} \times 4 = 20 \times 4$ .

$$x = 80,$$

$$5x = 25.$$

Divide both sides by 5 then  $\frac{5x}{5} = \frac{25}{5}$

$$x = 5.$$

When transposing numbers or letter symbols from one side to the other side multiplication becomes division and the division becomes multiplication.

The equality of an equation remains unchanged when both sides of the equation are treated in the same way. When transposing from one side to the other side,

a plus quantity becomes minus quantity.

a minus quantity becomes a plus quantity

a multiplication becomes a division

a division becomes a multiplication.

To solve simple equations isolate the unknown quantity which is to be found on the left side of the equation.

### Example

• Solve for x if  $4x = 3(35 - x)$

$$4x = 105 - 3x \text{ (brackets removed)}$$

$$4x + 3x = 105 \text{ (By transposing } -3x \text{ on the right side to the left side)}$$

$$7x = 105$$

$$x = 15 \text{ (dividing both sides by 7)}$$

## Assignment

### Add

1  $14f - 2f + 5f - 7f + 9f$

2  $3xy + 5xy - 2xy + 8xy - 4xy$

3  $17xy - 4xy + 13 - xy - 6$

4  $2a + a + 3a + 6a - 5b$

5  $8c + 5c + 3c + 2c$

6  $14d + 3d + 25e + 2e$

7  $5p + 3r - r - 2p$

8  $8t + 12u - t - 14u$

9  $x - z + y + z$

10  $15a + 13a - 37a$

11  $17a - 4b - 7a + 3b$

12  $9c - 15e + 4c + 3e$

13  $13f + 40g - 16f + 7f + 2g - 17g$

14  $30x + 45y - 17x - 16y$

15  $8a + 3c - 6b - 5c + 4a + 8b$

16  $27i + 17k - 5l + 12i - 31k + 19l$

17  $230m + 472P - 320n - 75m + 180n - 141p$

18  $230m + 420s + 370y + 225m - 510y - 110s$

19  $45b + 25c + 18b + 40c$

20  $14d + 3d + 25e + 2e + e + d$

21  $15a - (4a + 3a - 5a)$

22  $5x + 3y - (2x - 5y)$

23  $(x + 2y + 3z) + (4x - y + z)$

24  $(2x + 5y) + (4x - 8z) + (15z - 6y) + (z - 2x)$

25  $(-2x + 3y - 3z) + (-6y - 5x + z)$

26  $(a - 3b + 4c) + (-7c - a + 4b)$

27  $(2x + 5y) + (4x - 8z) + (15z - 2y)$

### Subtract

1  $38xy - 25xy$

2 Subtract  $2a - 3b - c$  from  $3a - 2b + 4c$

3  $2a - 3(a - (a - b))$

### Add and Subtract

1  $230a + 420b + 370c + 225a - 510c - 110b$

2  $15d - (4d + 3d - 5d)$

3  $8x + 3z - 6y - 5z + 4x + 8y$

### Multiplication

1  $5yzx \times (-5ab)$

2  $3ax - 9b$

3  $2ab \times -7pq$

### Division

1  $\frac{10a}{2a}$

2  $-3ax \div -6x$

3  $15xy \div -5$

4  $-\frac{8ac}{2bc}$

5  $\frac{-5m \times -6n - 7p}{-28mn}$

6  $\frac{5a + 20}{7a + 28}$



## Algebra - Theory of indices, Algebraic formula, related problems

## Calculations involving powers

## Power : Concept

a.a.a... upto n times is =  $a^n$

a is the base, n is the exponent.

When a number, say 2 is multiplied by itself 4 times, we write it as  $2^4$  (two to the power of 4) and it is equal to  $2 \times 2 \times 2 \times 2 = 16$ .

The exponent denotes how many times the base number is multiplied by itself.

Powers with a positive base have a positive result.

Powers with a negative base and with an exponent that is even will have a positive result.

The sign

$$(+a)^n = a^n$$

$$(-a)^{2n} = a^{2n}$$

$$(2)^2 = 2 \times 2 = 4 \text{ and}$$

$$(-2)^2 = -2 \times -2 = +4 \text{ but}$$

$$(-2)^3 = -2 \times -2 \times -2 = -8$$

## Addition and subtraction of powers

Powers with the same base and exponents can be added or subtracted by addition or subtraction of the coefficients.

$$x.a^n + y.a^n = a^n(x + y)$$

$$x.a^n - y.a^n = a^n(x - y)$$

$$\text{Ex } .4x^2 + x^2 - 3x^2 = x^2(4 + 1 - 3) = 2x^2.$$

## Multiplication

Powers with the same bases are multiplied by involving the common base raised to the power of sum of the exponents.

$$a^m \times a^n = a^{m+n}.$$

$$2^3 \times 2^2 = 2^{3+2} = 2^5$$

$$(2 \times 2 \times 2) \times (2 \times 2) = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$$

$$8 \times 4 = 32.$$

Powers with the same exponent of different base numbers are multiplied by involving the product of the base numbers raised to the common exponent.

$$a^n \times b^n = (a \times b)^n$$

$$2^2 \times 3^2 = (2 \times 3)^2$$

$$2 \times 2 \times 3 \times 3 = 6 \times 6 = 36$$

## Division

Powers with like bases are divided by involving the base raised to the difference between the exponents.

$$\frac{a^m}{a^n} = a^{m-n}$$

$$\frac{2^3}{2^2} = 2^{3-2} = 2^1 = 2$$

$$\frac{2 \times 2 \times 2}{2 \times 2} = \frac{8}{4} = 2$$

Powers with the same exponents are divided by involving the quotient of the bases by the common exponent.

$$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$$

$$\frac{2^2}{3^2} = \left(\frac{2}{3}\right)^2 = \frac{2 \times 2}{3 \times 3} = \frac{4}{9}$$

Only like powers can be added or subtracted.

## Examples

(The exponent 1 is usually not written.)

$$a^1 = a$$

$$2^1 = 2$$

$$2a^2 + 3a^2 = 5a^2$$

(Any number raised to the power of 0 is 1.)

$$a^0 = 1$$

$$2^0 = 1$$

A number raised to a negative power corresponds to its reciprocal with the exponent's sign changed to +.

$$a^{-n} = \frac{1}{a^n}$$

$$2^{-2} = \frac{1}{2^2}$$

Powers are involved by multiplying the exponents.

$$(a^n)^m = a^{nm}$$

$$(2^2)^3 = 2^{2 \cdot 3} = 2^6$$

Powers can be transposed without affecting the result.

$$(a^n)^m = (a^m)^n$$

$$(2^2)^3 = (2^3)^2$$

$$(2 \times 2) \times (2 \times 2) \times (2 \times 2) = (2 \times 2 \times 2) (2 \times 2 \times 2)$$

$$4 \times 4 \times 4 = 64$$

$$8 \times 8 = 64$$

A mixed number raised to a power is first converted into an improper fraction and then the result is evaluated.



$$8 \quad 3x^3y^2 \div xy$$

$$= \frac{3x^3y^2}{xy} = 3x^2y$$

9 Divide  $45a^2b^2c$  by  $9a^2c$

$$= \frac{45a^2b^2c}{9a^2c}$$

$$= 5b^2$$

### Algebraic Formulae

1	$(a + b)^2$	$= a^2 + b^2 + 2ab$
2	$(a - b)^2$	$= a^2 + b^2 - 2ab$
3	$(a + b)^2$	$= (a - b)^2 + 4ab$
4	$(a - b)^2$	$= (a + b)^2 - 4ab$ ; $(a + b)^2 - (a - b)^2 = 4ab$
5	$a^2 + b^2$	$= (a + b)^2 - 2ab = (a - b)^2 + 2ab$
6	$a^2 - b^2$	$= (a + b)(a - b)$
7	$a^3 + b^3$	$= (a + b)(a^2 + b^2 - ab)$
8	$a^3 - b^3$	$= (a - b)(a^2 + b^2 + ab)$
9	$(a + b)^3$	$= a^3 + b^3 + 3ab(a + b)$
10	$(a - b)^3$	$= a^3 - b^3 - 3ab(a - b)$
11	$(a + b + c)^2$	$= a^2 + b^2 + c^2 + 2(ab + bc + ca)$
12	$a^4 - b^4$	$= (a^2 + b^2)(a + b)(a - b)$

### Examples

1 If  $x + y = 9$  and  $xy = 20$

Find i)  $x^2 + y^2$     ii)  $x - y$     iii)  $x^2 - y^2$   
 iv)  $x^3 + y^3$     v)  $x^3 - y^3$     vi)  $x$  and  $y$

**i**  $(a + b)^2 = a^2 + b^2 + 2ab$

$$(x + y)^2 = x^2 + y^2 + 2xy$$

$$(9)^2 = x^2 + y^2 + 2(20)$$

$$81 = x^2 + y^2 + 40$$

$$x^2 + y^2 = 81 - 40$$

$$x^2 + y^2 = 41$$

**ii**  $(a - b)^2 = (a + b)^2 - 4ab$

$$(x - y)^2 = (x + y)^2 - 4xy$$

$$= (9)^2 - 4(20)$$

$$= 81 - 80$$

$$= 1$$

$$x - y = \sqrt{1} = 1$$

**iii**  $a^2 - b^2 = (a + b)(a - b)$

$$x^2 - y^2 = (x + y)(x - y)$$

$$= 9 \times 1$$

$$x^2 - y^2 = 9$$

**iv**  $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$$x^3 + y^3 = (x + y)(x^2 + y^2 - xy)$$

$$= 9(41 - 20)$$

$$= 9 \times 21$$

$$x^3 + y^3 = 189$$

**v**  $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$$x^3 - y^3 = (x - y)(x^2 + y^2 + xy)$$

$$= 1(41 + 20)$$

$$= 1 \times 61$$

$$= 61$$

$$x^3 - y^3 = 61$$

**vi**  $x + y = 9$   
 $x - y = 1$

---


$$2x = 10$$


---

$$x = \frac{10}{2} = 5$$

If  $x = 5$ ,  $5 + y = 9$   
 $y = 9 - 5 = 4$

$$x = 5; y = 4$$

2 Solve  $(x + 5)^2 - (x - 5)^2$

If  $x + 5 = a$  and  $x - 5 = b$

$$a^2 - b^2 = (a + b)(a - b)$$

$$(x + 5)^2 - (x - 5)^2 = [(x + 5) + (x - 5)][(x + 5) - (x - 5)]$$

$$= (x + 5 + x - 5)(x + 5 - x + 5)$$

$$= (2x)(10)$$

$$= 20x$$

3 If  $(x - y) = 4$  and  $xy = 12$ , find the value of  $(x^2 + y^2)$

$$(x - y)^2 = x^2 + y^2 - 2xy$$

$$(4)^2 = x^2 + y^2 - 2 \times 12$$

$$16 = x^2 + y^2 - 24$$

$$x^2 + y^2 - 24 = 16$$

$$x^2 + y^2 = 16 + 24$$

$$x^2 + y^2 = 40$$

4 If  $x - y = 7$  and  $xy = 60$  then find the value of  $x^4 + y^4$

$$(x - y)^2 = x^2 + y^2 - 2xy = 7^2$$

$$x^2 + y^2 - 2 \times 60 = 49$$

$$x^2 + y^2 = 169$$

$$(x^2 + y^2)^2 = (169)^2 \text{ (take square on both side)}$$

$$x^4 + y^4 + 2x^2y^2 = (169)^2$$

$$x^4 + y^4 + 2(xy)^2 = 28561$$

$$x^4 + y^4 + 2(60)^2 = 28561$$

$$x^4 + y^4 + 2(3600) = 28561$$

$$x^4 + y^4 + 7200 = 28561$$

$$x^4 + y^4 = 28561 - 7200$$

$$x^4 + y^4 = 21361$$

5  $x + y = \sqrt{5}$ ;  $x - y = \sqrt{3}$  Find the value of  $8xy(x^2 + y^2)$

$$x + y = \sqrt{5}; x - y = \sqrt{3} \text{ (take square on both sides)}$$

$$(x + y)^2 = 5; (x - y)^2 = 3$$

Solve the equations

$$(x + y)^2 = x^2 + y^2 + 2xy = 5$$

$$(x - y)^2 = x^2 + y^2 - 2xy = 3$$

---


$$2(x^2 + y^2) = 8$$

$$(x^2 + y^2) = \frac{8}{2} = 4$$

$$= x^2 + y^2 + 2xy = 5$$

$$= x^2 + y^2 - 2xy = 3$$

$$\begin{array}{cccc} (-) & (-) & (+) & (-) \end{array}$$

---


$$4xy = 2$$

$$xy = \frac{2}{4} = \frac{1}{2}$$

$$8xy(x^2 + y^2) = 8 \times \frac{1}{2} \times 4$$

$$= 4 \times 4 = 16$$

6 If  $(a - \frac{1}{a}) = 6$ . Find the value of  $a^2 + \frac{1}{a^2}$

$$\left(a - \frac{1}{a}\right) = 6$$

$$\left(a - \frac{1}{a}\right)^2 = 6^2 \text{ (take square on both sides)}$$

$$a^2 + \left(\frac{1}{a}\right)^2 - 2(a)\left(\frac{1}{a}\right) = 36$$

$$a^2 + \frac{1}{a^2} - 2 = 36$$

$$a^2 + \frac{1}{a^2} = 36 + 2$$

$$a^2 + \frac{1}{a^2} = 38$$

7 If  $x - \frac{1}{x} = 2$ , Find the value of  $x^3 - \frac{1}{x^3}$

$$(a - b)^3 = a^3 - b^3 - 3ab(a - b)$$

$$\left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3(x)\left(\frac{1}{x}\right)\left(x - \frac{1}{x}\right)$$

$$= x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right)$$

$$2^3 = x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right)$$

$$8 = x^3 - \frac{1}{x^3} - 3(2)$$

$$8 = x^3 - \frac{1}{x^3} - 6$$

$$8 + 6 = x^3 - \frac{1}{x^3}$$

$$14 = x^3 - \frac{1}{x^3}$$

$$x^3 - \frac{1}{x^3} = 14$$

8 If  $x - \frac{1}{x} = 4$ , Find the value of  $x^4 + \frac{1}{x^4}$

$$x - \frac{1}{x} = 4 \text{ (take square on both sides)}$$

$$\left(x - \frac{1}{x}\right)^2 = 4^2 [(a - b)^2 = a^2 + b^2 - 2ab]$$

$$x^2 + \frac{1}{x^2} - 2 \times x \times \frac{1}{x} = 4^2$$

$$x^2 + \frac{1}{x^2} - 2 = 16$$

$$x^2 + \frac{1}{x^2} = 16 + 2$$

$$x^2 + \frac{1}{x^2} = 18$$

$$\left(x^2 - \frac{1}{x^2}\right)^2 = (18)^2 \text{ (take square on both sides)}$$

$$(x^2)^2 + \left(\frac{1}{x^2}\right)^2 + 2 \times x^2 \times \frac{1}{x^2} = 324$$

$$x^4 + \frac{1}{x^4} + 2 = 324$$

$$x^4 + \frac{1}{x^4} = 324 - 2$$

$$x^4 + \frac{1}{x^4} = 322$$

## Assignment

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### Add

1  $(5x^2 - 3y^2 + z) + (-x^2 + 2y^2 - 4z)$

2  $7a^2 - 5a^2 + a^2 + 3a^2$

3  $3m^2n - 2m^2n + 4m^2n - m^2n + 7m^2n$

4  $18 + 13x^2 - 13 + 2x^2 - 15x^2$

5  $6l^2m + 3lm^2 - 2l^2m - 17lm^2 + 1$

6  $3a^2b - 2ab - 2a^2b - 3ab - 2a^2b + ab$

### Subtract

1 Subtract  $2a^2 - 3b^2$  from  $3a^2 + 2b^2$

2 Subtract  $-2y^2 + 3xy - 5$  from  $3x^2 - 4xy + 7y^2 - 5$

3 Subtract  $3x - 4x^2 + 2y^2$  from  $4y^2 - 2x + 8x^2$

### Add and Subtract

1  $48m^2 + 24m^2n + 12m^2 - 6m^2 - 12m^2n$

2  $3x^2y - 2xy - 2x^2y - 3xy - 2x^2y + xy$

3  $10x + 14 - 7y^2 - 11a + 2x - 4 - 3y^2 - 4a + 8$

### Multiplication

1  $7pq^2 \times 5r$

2  $(4x^2 + 3y^2) \times (-2z)$

3  $-7p \times 4q^2$

4  $p^2q^3 \times 3p^3q^2$

5  $(3b^2 - 2b)3b^2$

6  $5y \times 2y^3y^2$

7  $ab^{-1} \times ba^{-1}$

### Division

1  $4a^8 \div 2a^3$

2  $-15a^8 \div 3a^5$

3  $\frac{8a^4}{12a^{-7}}$

4  $\frac{3p^2 \times 4p \times 5p^3 \times p}{6p^4 \times 10p}$

5  $\frac{25m^2n}{5m^3n^2}$

## Profit and loss - Simple problems on profit & loss

### Definition of 'profit and loss statement (P&L)

A profit and loss statement (P&L) is a financial statement that summarizes the revenues, costs and expenses incurred during a specific period of time, usually a year. These records provide information about a company's ability - to generate profit by increasing revenue, reducing costs, or both. The P&L statement is also referred to as "statement of profit and loss", "income statement", "statement of operations", "statement of financial results" and "income and expenditure statement".

### Profit and loss

#### Important facts

#### Cost price

The price, at which an article is purchased is called its cost price, abbreviated as C.P.

#### Selling price

The price at which an article is sold, is called its selling price, abbreviated as S.P.

#### Profit or gain

If S.P. is greater than C.P., the seller is said to have a profit or gain.

#### Loss

If S.P. is less than C.P., the seller is said to have incurred a loss.

#### Discount

The reduction given to the selling price of a product is the discount.

#### Important formulae

1 Profit or Gain = (S.P.) - (C.P.)

2 Loss = (C.P.) - (S.P.)

3 Loss or gain always depends on C.P.

4 Profit/gain is always expressed in %.

$$\text{Gain\%} = \left( \frac{\text{Gain} \times 100}{\text{C.P.}} \right)$$

5 Loss percentage: (Loss %)

$$\text{Loss \%} = \left( \frac{\text{Loss} \times 100}{\text{C.P.}} \right)$$

6 Selling price: (S.P)

$$\text{SP} = \left( \frac{100 + \text{Gain\%}}{100} \times \text{C.P.} \right)$$

7 Selling price: (S.P)

$$\text{SP} = \left( \frac{(100 - \text{loss \%})}{100} \times \text{C.P.} \right)$$

8 Cost price: (C.P)

$$\text{C.P.} = \left( \frac{100}{(100 + \text{Gain \%})} \times \text{S.P.} \right)$$

9 Cost price: (C.P)

$$\text{C.P.} = \left( \frac{100}{(100 - \text{Loss \%})} \times \text{S.P.} \right)$$

10 If an article is sold at a gain of say 35%, then S.P. = 135% of C.P.

11 If an article is sold at a loss of say, 35% then S.P. = 65% of C.P.

### Example

**1 A dealer bought a television set for Rs.10,000 and sold it for Rs.12,000. Find the profit made by him for 1 television set. If he had sold 5 television sets, find the total profit?**

#### Solution

Selling price of the television set = Rs.12,000

Cost price of the television set = Rs.10,000

S.P. > C.P., there is a profit

Profit = S.P. - C.P.

= 12000 - 10000

Profit = Rs.2,000

Profit on 1 television set = Rs.2000

Profit on 5 television sets = 2000 x 5

= **Rs.10,000**

**2 Sanjay bought a bicycle for Rs.5000. He sold it for Rs.600 less after two years. Find the selling price and the loss percent?**

#### Solution

Cost price of the bicycle = Rs.5000

Loss = Rs.600

Selling price = Cost price - loss

= 5000 - 600

Selling price of the bicycle = Rs.4400

Loss % =  $\frac{\text{Loss}}{\text{C.P.}} \times 100$

=  $\frac{600}{5000} \times 100$

**Loss = 12%**

**3 A man bought an old bicycle for Rs.1250. he spent Rs.250 on its repairs. He then sold it for Rs.1400. Find his loss %?**

#### Solution

Cost price of the bicycle = Rs.1250

Repair Charges = Rs.250  
 Total cost price = 1250+250 = Rs.1500  
 Selling price = Rs.1400  
 C.P > S.P, there is a loss

$$\text{Loss} = \text{Cost price} - \text{Selling price}$$

$$1500 - 1400 = 100$$

$$\text{Loss} = \text{Rs.}100$$

$$\text{Loss \%} = \frac{\text{Loss}}{\text{C.P.}} \times 100$$

$$= \frac{100}{1500} \times 100$$

$$= \frac{20}{3} = 6\frac{2}{3}\% \text{ (or) } 6.67\%$$

$$\text{Loss} = 6.67\%$$

**Profit percentage or loss percentage is always calculated on the cost price of the article.**

**4 A fruit seller bought 8 boxes of grapes at Rs.150 each. One box was damaged. He sold the remaining boxes at Rs.190 each. Find the profit percent?**

**Solution**

Cost price of 1 box of grapes = Rs.150  
 Cost price of 8 boxes of grapes = 150 x 8  
 = Rs.1200  
 Number of boxes damaged = 1  
 Number of boxes sold = 8 - 1 = 7  
 Selling price of 1 box of grapes = Rs.190  
 Selling price of 7 boxes of grapes = 190 x 7  
 = Rs.1330

S.P.>C.P, there is a profit

$$\text{Profit} = \text{Selling price} - \text{Cost price}$$

$$= 1330-1200$$

$$= 130$$

$$\text{Profit} = \text{Rs.}130$$

$$\text{Percentage of profit} = \frac{\text{Profit}}{\text{C.P}} \times 100$$

$$= \frac{130}{1200} \times 100$$

$$= 10.83$$

$$\text{Profit} = 10.83\%$$

**5 Ram, the shopkeeper bought a pen for Rs.50 and then sold it at a loss of Rs.5. Find his selling price.**

**Solution**

$$\text{Cost price of the pen} = \text{Rs.}50$$

$$\text{Loss} = \text{Rs.}5$$

$$\text{S.P.} = \text{C.P.} - \text{Loss}$$

$$= 50 - 5 = 45$$

$$\text{Selling price of the pen} = \text{Rs.}45$$

**6 Find the initial amount if 12% of the total amount it is ₹ 1080**

Let the initial amount be 'x'

$$\text{Given: } 12\% \text{ of the total amount} = \text{Rs.}1080$$

$$\frac{12}{100} \times x = 1080$$

$$x = \frac{1080 \times 100}{12}$$

$$= ₹ 9000$$

$$\therefore \text{The initial amount} = \text{Rs.}9000$$

**Applications of profit and loss**

In this section, we learn to solve problems on applications of profit and loss.

**i Illustration of the formula for S.P.**

Consider the following situation

Rajesh buys a pen for Rs.80 and sells it to his friend.

If he wants to make a profit of 5%, can you say the price for which he would have sold?

(Rajesh bought the pen for Rs.80 which is the cost price (C.P.). When he sold, he makes a profit of 5% which is calculated on the C.P.)

$$\therefore \text{Profit} = 5\% \text{ of C.P.} = \frac{5}{100} \times 80 = \text{Rs.}4$$

Since there is a profit, S.P > C.P.

$$\text{S.P.} = \text{C.P.} + \text{Profit}$$

$$= 80 + 4 = \text{Rs.}84$$

**\therefore The price for which Rajesh would have sold = Rs.84**

**The same problem can be done using the formula.**

$$\text{Selling price (S.P)} = \frac{(100 + \text{Profit \%})}{100} \times \text{C.P}$$

$$= \frac{(100 + 5)}{100} \times 80$$

$$= \frac{105}{100} \times 80 = \text{Rs.}84$$

**ii Illustration of the formula for C.P**

Consider the following situation

Suppose a shopkeeper sells a wrist watch for Rs. 540 making a profit of 5%, then what would have been the cost of the watch?

(The shopkeeper had sold the watch at a profit of 5% on the C.P. Since C.P. is not known, let us take it as Rs. 100)

Profit of 5% is made on the C.P.

$$\therefore \text{Profit} = 5\% \text{ of C.P.}$$

$$= \frac{5}{100} \times 100 = \text{Rs. } 5$$

$$\text{We know S.P.} = \text{C.P.} + \text{Profit}$$

$$= 100 + 5$$

$$= \text{Rs. } 105$$

Here, if S.P. is Rs.105, then C.P. is Rs. 100

$$\text{When S.P. of the watch is Rs. } 540, \text{ C.P.} = \frac{540 \times 100}{105}$$

$$= \text{Rs. } 514.29$$

$\therefore$  The watch would have cost Rs. 514.29 for the shopkeeper.

The above problem can also be solved by using the formula method.

$$\text{C.P.} = \left( \frac{100}{100 + \text{Profit}\%} \right) \times \text{S.P.}$$

$$= \left( \frac{100}{100 + 5} \right) \times 540$$

$$= \frac{100}{105} \times 540 = \text{Rs. } 514.29$$

We now summarize the formulae to calculate S.P. and C.P. as follows.

<p><b>1 When there is a profit</b></p> $\text{C.P.} = \left( \frac{100}{100 + \text{Profit}\%} \right) \times \text{S.P.}$	<p><b>1 When there is a loss</b></p> $\text{C.P.} = \left( \frac{100}{100 - \text{Loss}\%} \right) \times \text{S.P.}$
<p><b>2 When there is a profit</b></p> $\text{S.P.} = \left( \frac{100 + \text{Profit}\%}{100} \right) \times \text{C.P.}$	<p><b>2 When there is a loss</b></p> $\text{S.P.} = \left( \frac{100 - \text{Loss}\%}{100} \right) \times \text{C.P.}$

### Example

**1 Hameed buys a colour T.V. set for Rs. 15,200 and sells it at a loss of 20%. What is the selling price of the T.V. set?**

$$= \frac{80}{100} \times 15200$$

$$= \text{Rs. } 12,160$$

### Solution

#### Method - I

Loss is 20% of the C.P.

$$\frac{20}{100} \times 15200 = \text{Rs. } 3040$$

$$\text{S.P.} = \text{C.P.} - \text{Loss}$$

$$15200 - 3040 = \text{Rs. } 12160$$

#### Method - II

$$\text{C.P.} = \text{Rs. } 15,200$$

$$\text{Loss} = 20\%$$

$$\text{S.P.} = \frac{100 - \text{Loss}\%}{100} \times \text{C.P.}$$

$$= \frac{100 - 20}{100} \times 15200$$

**2 A scooty is sold for Rs. 13600 and fetches a loss of 15%. Find the cost price of the scooty.**

#### Method - I

Loss of 15% means,

$$\text{If C.P. is Rs. } 100, \text{ Loss} = \text{Rs. } 15$$

Therefore, S.P. would be

$$(100 - 15) = \text{Rs. } 85$$

If S.P. is Rs. 85, C.P. is Rs. 100

When S.P. is Rs. 13600 then

$$\text{C.P.} = \frac{100 \times 13600}{85} = \text{Rs. } 16000$$

#### Method - II

$$\text{Loss} = 15\%$$

$$\text{S.P.} = \text{Rs. } 13600$$

$$\text{C.P.} = \left( \frac{100}{100 - \text{Loss}\%} \right) \times \text{S.P.}$$



$$= \frac{100}{100 - 15} \times 13600$$

$$= \frac{100}{85} \times 13600$$

$$= \text{Rs. } 16000$$

Hence the cost price of the scotty is Rs. 16000

### Discount

Discount is the reduction in value on the marked price or list price of the article.

The market price of a product is Rs.550

Amount paid by pooja to the shop keeper is Rs. 440

$$\begin{aligned} \text{Discount} &= \text{Rs. } 550 - \text{Rs. } 440 \\ &= \text{Rs. } 110 \\ &= \text{Marked price} - \text{Selling price} \end{aligned}$$

Hence we conclude the following

$$\begin{aligned} \text{Discount} &= \text{Marked price} - \text{Selling price} \\ \text{Selling price} &= \text{Marked price} - \text{Discount} \\ \text{Marked price} &= \text{Selling price} + \text{Discount} \end{aligned}$$

### Example

**1 A bicycle marked at Rs. 1500 is sold for Rs. 1350. What is the percentage of discount?**

Marked price = Rs. 1500

Selling price = Rs. 1350

$$\begin{aligned} \text{Amount of discount} &= \text{Marked price} - \text{Selling price} \\ &= 1500 - 1350 \\ &= \text{Rs. } 150 \end{aligned}$$

Discount for Rs. 1500 = Rs. 150

$$\text{Discount for Rs. } 100 = \frac{150}{1500} \times 100$$

Percentage of discount = 10%

**2 The list price of a Frock is Rs.220. A discount of 20% on sales is announced. What is the amount of discount on it and its selling price?**

$$\text{Amount of discount} = \frac{\text{Discount}}{100\%} \times \text{M.P.}$$

$$\text{Amount of discount} = \frac{20}{100} \times 220 = \text{Rs. } 44$$

Selling price of the frock = Marked price - Discount

$$220 - 44 = \text{Rs. } 176$$

**3 An almirah is sold at Rs. 5225 after allowing a discount of 5%. Find its marked price.**

### Solution

#### Method - I

The discount is given in percentage

Hence, the M.P. is taken as Rs. 100

Rate of discount = 5%

$$\text{Amount of discount} = \frac{5}{100} \times 100$$

$$\begin{aligned} \text{Selling price} &= \text{M.P.} - \text{Discount} \\ &= 100 - 5 = \text{Rs. } 95 \end{aligned}$$

If S.P. is Rs. 95, then M.P. is Rs.100

When S.P. is Rs. 5225

$$\text{M.P.} = \frac{100}{95} \times 5225$$

M.P of the almirah = Rs. 5500

#### Method - II

S.P = Rs. 5225

Discount = 5%

M.P = ?

$$\text{M.P} = \left( \frac{100}{100 - \text{Discount}\%} \right) \times \text{S.P.}$$

$$= \left( \frac{100}{100 - 5} \right) \times 5225$$

= Rs. 5500

**4 A shopkeeper allows a discount of 10% to his customers and still gains 20%. Find the marked price of an article which costs Rs.450 to the shopkeeper.**

### Solution

#### Method - I

Let M.P be Rs. 100

Discount = 10% of M.P

$$= \frac{10}{100} \text{ of M.P} = \frac{10}{100} \times 100$$

= Rs. 10

S.P = M.P - Discount

= 100 - 10

= Rs. 90

Gain = 20% of C.P.

$$= \frac{20}{100} \times 450 = \text{Rs. } 90$$

S.P = C.P + Gain

= 450 + 90 = Rs. 540

If S.P. is Rs. 90, then M.P is Rs. 100

$$\text{M.P.} = \frac{540 \times 100}{90} = \text{Rs. } 600$$

The M.P. of an article = Rs. 600

#### Method - II

Discount = 10%, Gain = 20%

C.P. = Rs. 450, M.P. = ?

$$\begin{aligned}\text{M.P.} &= \frac{100 + \text{Gain}\%}{100 - \text{Discount}\%} \times \text{C.P.} \\ &= \frac{(100 + 20)}{(100 - 10)} \times 450 \\ &= \frac{120}{90} \times 450 \\ &= \text{Rs. } 600\end{aligned}$$

**5 A dealer allows a discount of 10% and still gains 10%. What is the cost price of the book which is marked at Rs. 220?**

#### Solution

#### Method - I

$$\begin{aligned}\text{M.P.} &= \text{Rs. } 220 \\ \text{Discount} &= 10\% \text{ of M.P.} \\ &= \frac{10}{100} \times 220 \\ &= \text{Rs. } 22 \\ \text{S.P.} &= \text{M.P.} - \text{Discount} \\ &= 220 - 22 \\ &= \text{Rs. } 198\end{aligned}$$

Let, C.P. be Rs. 100

$$\begin{aligned}\text{Gain} &= 10\% \text{ of C.P.} \\ &= \frac{10}{100} \times 100 \\ &= \text{Rs. } 10 \\ \text{S.P.} &= \text{C.P.} + \text{Gain} \\ &= 100 + 10 \\ &= \text{Rs. } 110\end{aligned}$$

If S.P. is Rs. 110, then C.P is Rs. 100

When S.P. is Rs. 198,

$$\begin{aligned}&= \frac{198 \times 100}{110} \\ &= \text{Rs. } 180\end{aligned}$$

#### Method - II

$$\begin{aligned}\text{Discount} &= 10\% \\ \text{Gain} &= 10\% \\ \text{M.P.} &= \text{Rs. } 220 \\ \text{C.P.} &= \frac{100 - \text{Discount}\%}{100 + \text{Gain}\%} \times \text{M.P.} \\ &= \frac{100 - 10}{100 + 10} \times 220 \\ &= \frac{90}{110} \times 220 \\ &= \text{Rs. } 180\end{aligned}$$

**6 A trader buys an article for Rs. 1200 and marks it 30% above the C.P. He then sells it after allowing a discount of 20%. Find the S.P. and profit percent.**

#### Solution

Let C.P. of the article be Rs. 100

M.P. = 30% above C.P. = Rs. 130

If C.P is Rs. 100, then M.P. is Rs. 130

When C.P. is Rs. 1200,

$$\text{M.P.} = \frac{1200 \times 130}{100} = \text{Rs. } 1560$$

$$\text{Discount} = 20\% \text{ of } 1560 = \frac{20}{100} \times 1560$$

$$\text{Discount} = 20\% \text{ of } 1560 = \frac{20}{100} \times 1560$$

$$= \text{Rs. } 312$$

$$\begin{aligned}\text{S.P.} &= \text{M.P.} - \text{Discount} \\ &= 1560 - 312 \\ &= \text{Rs. } 1248\end{aligned}$$

$$\begin{aligned}\text{Profit} &= \text{S.P.} - \text{C.P.} \\ &= 1248 - 1200 \\ &= \text{Rs. } 48\end{aligned}$$

$$\text{Profit \%} = \frac{\text{Profit}}{\text{C.P.}} \times 100$$

$$= \frac{48}{1200} \times 100$$

$$= 4\%$$

## Summary

Percent means per hundred. A fraction with its denominator 100 is called a percent.

**In case of profit, we have Profit = S.P - C.P.**

**In case of loss, we have Loss = C.P - S.P.**

$$\text{Profit \%} = \frac{\text{Profit}}{\text{C.P.}} \times 100$$

$$\text{Loss \%} = \frac{\text{Loss}}{\text{C.P.}} \times 100$$

$$\text{S.P.} = \left( \frac{100 + \text{Profit\%}}{100} \right) \times \text{C.P.}$$

$$\text{S.P.} = \left( \frac{100 - \text{Loss\%}}{100} \right) \times \text{C.P.}$$

$$\text{C.P.} = \left( \frac{100}{100 + \text{Profit\%}} \right) \times \text{S.P.}$$

$$\text{C.P.} = \left( \frac{100}{100 - \text{Loss\%}} \right) \times \text{S.P.}$$

$$\text{M.P.} = \frac{100}{100 - \text{Discount \%}} \times \text{S.P.}$$

$$\text{S.P.} = \frac{100 - \text{Discount \%}}{100} \times \text{M.P.}$$

$$\text{C.P.} = \frac{100 - \text{Discount \%}}{100 + \text{Profit \%}} \times \text{M.P.}$$

$$\text{M.P.} = \frac{100 + \text{Profit \%}}{100 - \text{Discount \%}} \times \text{C.P.}$$

$$\text{Discount percent} = \frac{\text{Discount}}{\text{M.P.}} \times 100$$

Discount is the reduction given on the Marked price.

Selling price is the price payable after reducing the discount from the marked price.

Discount = M.P. - S.P.

## Assignment

- 1 Find the cost price if the product is sold at Rs. 572 with a profit of Rs. 72.
- 2 Find the C.P if the product is sold at Rs.1973 with a profit of Rs. 273
- 3 Find the selling price if the cost price is Rs. 7282 with a profit of Rs. 208
- 4 Find out the S.P. if the C.P. is Rs. 9684 with a loss of Rs. 684
- 5 Find out the profit percentage if the C.P is Rs. 320 and S.P is Rs. 384.
- 6 Find out the profit amount if the C.P. and S.P. are Rs. 2500 and Rs. 2700 respectively.
- 7 Calculate the percentage of loss if the C.P. and S.P are Rs. 40 and Rs. 38 respectively.
- 8 A computer table bought at Rs. 1150 with Rs. 50 as a transport charge. Calculate the S.P. if the profit is of 5%
- 9 By selling a table for Rs. 1320 with a gain of 10%. Find the C.P.
- 10 The C.P. of 16 bolts is equal to the S.P. of 12 bolts. Find the gain percent.

## Estimation and Costing - Simple estimation of the requirement of material etc., as applicable to the trade - Calculate the cost of filing a plot with construction soil

Estimation is the method of calculating the various quantities and the expenditure to be incurred on a particular job or process.

Estimate is the method used to measure or quantify the different quantities and the expected expenditure to be incurred on a particular work or project.

We know that the estimation is long procedure, and its totally depends upon the projects,

In case the funds available are less than the estimated cost the work is done in part or by reducing it or specifications are altered,

The following essential details are required for preparing an estimate.

Drawings like plan, elevation and sections of important parts.

Detailed specifications about workmanship & properties of materials, etc.

Standard schedule of rates of the current year.

Estimating is the process of preparing an approximation of quantities which is a value used as input data and it is derived from the best information available.

An estimate that turns out to be incorrect will be an overestimate if the estimate exceeded the actual result, and an underestimate if the estimate fell short of the actual result.

A cost estimate contains approximate cost of a product process or operation. The cost estimate has a single total value and it is inclusive of identifiable component values.

### Purpose of Estimating and Costing

- 1 Estimates provide an rough idea of the cost of the job and therefore its feasibility can be calculated, i.e. whether or not the project would be included in the funds available.
- 2 Estimate gives an idea of the time needed to complete the work.
- 3 Estimates are required to invite tenders and quotations and to arrange the contracts.
- 4 Estimates are also required to control expenditure during the execution of the work.

- 5 Estimates decide whether or not proposed plan matches the funds available.

### Estimation Methods

Estimate involves the following operations

- Preparing detailed Estimate.
- Calculating the rate of each unit of work.
- Preparing abstract of estimate.

**Estimation** is the process of calculating or evaluating a quantity by estimation, that is, without reference to specific measurements. Estimating is a fundamental process in all engineering.

This is usually done before purchase or construction begins or during preliminary planning stages. Estimating is usually more accurate, but there are a few limitations - namely that if your estimate relies on labour costs, you'll need to know how many man-hours will take to complete the project.

Estimates are developed from observations and knowledge of past experience. The accuracy of an estimate often depends on the level of detail available and the amount of time for which data are available for analysis.

**Costing** is the process of estimating the cost of a project before it's completed. It can be done with an itemized list, or through estimation using a construction cost calculator.

Costing includes three steps: estimating, bidding, and finalizing. It helps predict how much money will be required to construct the project.

A "costing" typically refers to how much it will cost someone to produce a single unit.

### There are two types of costings

**Independent costing** - this is the cost of direct material and labour costs. This type of costing only takes into account the cost of a single-phase, so it's not representative of the overall project cost.

**Cumulative Costing** - this type of costing looks at the total cost for all phases of work, but it can be difficult to ensure that estimates are accurate.

**Exercise: Calculate the cost of filing a plot with construction soil**

Let us consider a plot or site as shown in the drawing for the calculation purpose.

Length of plot = 30 m

Breadth of the plot = 20 m

Plot to be filled at front end = 0.75 m

Plot to be filled at back end = 1.25 m

The volume of the plot to be filled with construction soil

$$\begin{aligned} &= l \times b \times (\text{Average depth of the plot}) \\ &= 30 \times 20 \times \{(0.75 + 1.25) \div 2\} \\ &= 30 \times 20 \times 1 \\ &= 600 \text{ m}^3 \end{aligned}$$

**No of truck required**

The volume of soil required ÷ volume of a truck

Aproximately

$$\begin{aligned} \text{Soil} &= 600\text{m}^3 \div \text{Truck volume } 30\text{m}^3 \\ &= \frac{600}{30} = 20 \text{ truck} \end{aligned}$$

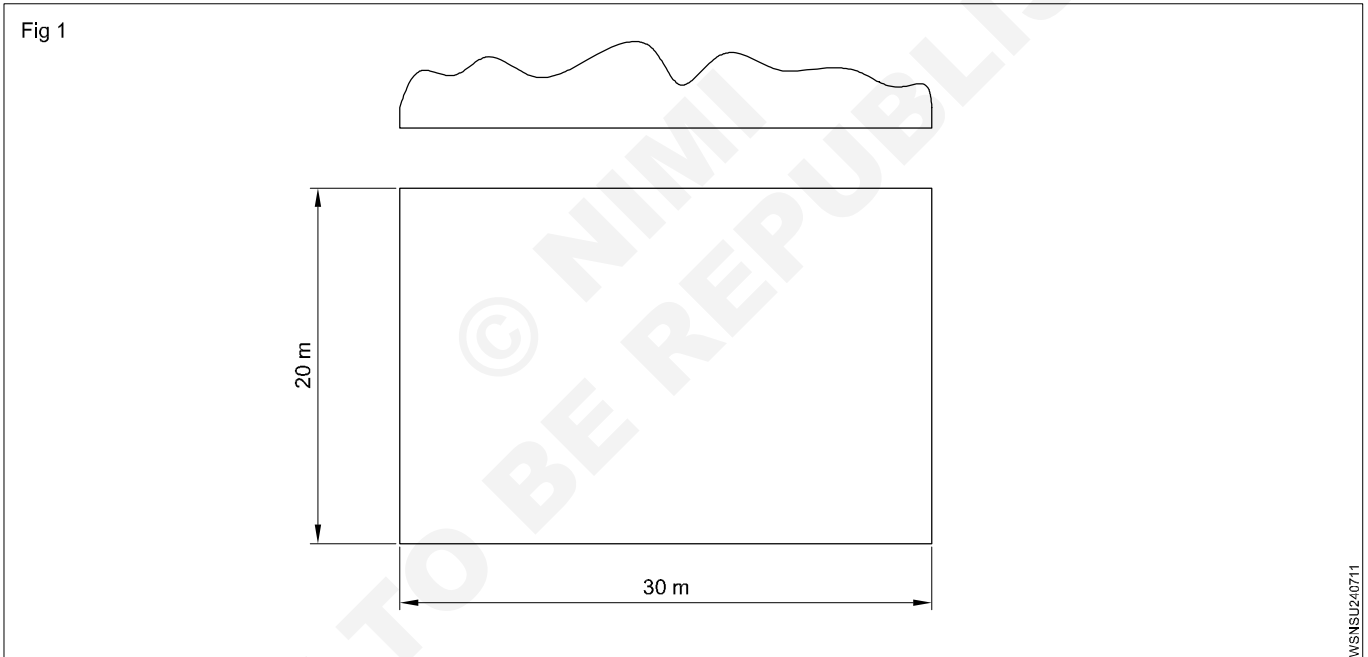
**The cost of filing**

The cost of construction soil varies according to the regional market rate.

The cost of construction soil in a range of Rs.3000 to Rs.4000 per cum.

Let us take Rs.3000 for calculation purpose

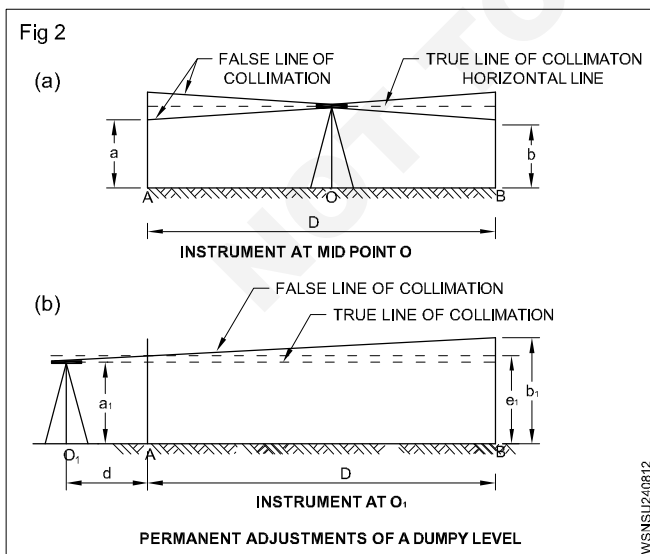
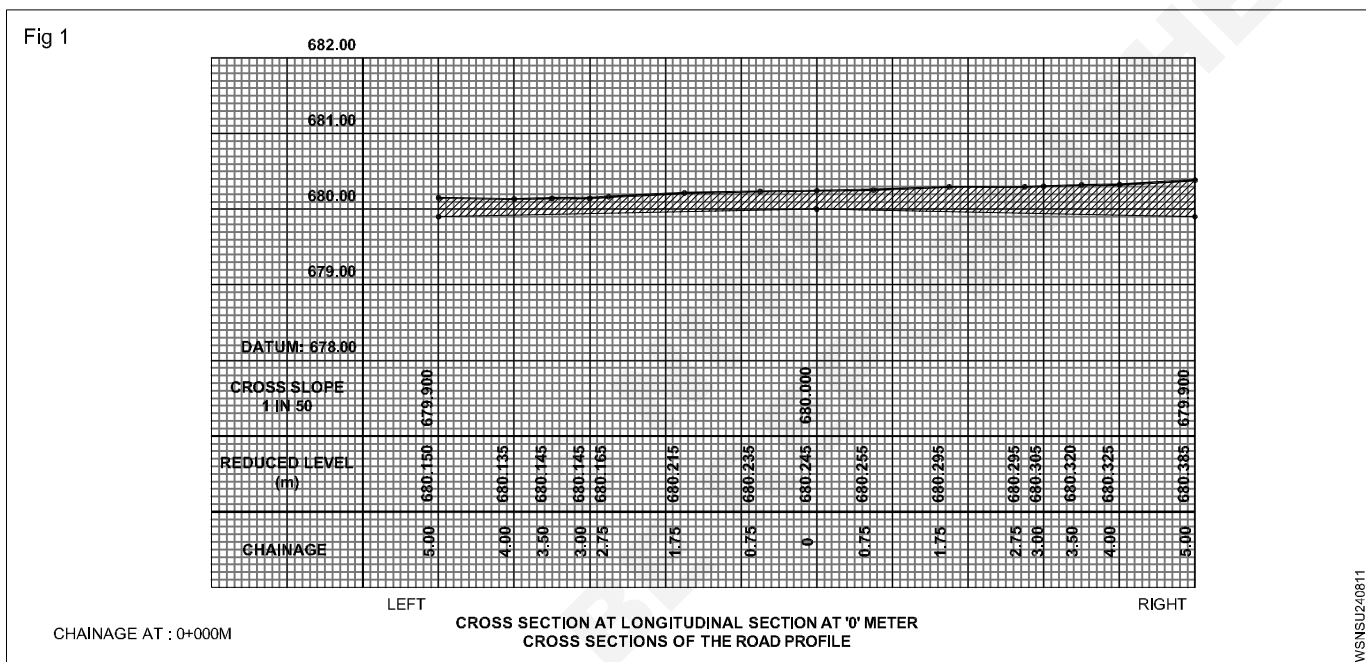
$$\begin{aligned} \text{Cost of filling} &= \text{No. of trucks} \times \text{Cost of volume of trucks} \\ &= 20 \times 3000 \\ &= \text{Rs.60,000} \end{aligned}$$



**Estimation and Costing - Simple estimation of the requirement of material etc., as applicable to the trade - Practice profile levelling or longitudinal and cross section levelling - plotting profile**

Practice profile levelling or longitudinal and cross section levelling - plotting profile.

- 1 Calculate the levels according to 1 in 50 from chainage 10 to chainage 200. With reduced level at '0' chainage 680.00
- 3 Mark a point at 680.000 at ... on Y- Axis
- 4 Similarly scale of other formation levels on respective chainage points.
- 5 Join these points with straight line to get the formation line.
- 6 Draw the formation line with red ink.



Chainage	GroundLevel	Formation Level	Depth of Cutting	Height of Filling
0	680.245	680.00	0.245	
10	680.335	680.5		0.165
20	680.395	680.30	0.090	
30	680.525	680.45	0.075	
40	680.665	680.60	0.065	
50	680.775	680.75	0.025	
60	680.965	680.90	0.065	
70	681.210	681.05	0.160	
80	681.370	681.20	0.170	
90	681.645	681.35	0.295	
100	681.840	681.50	0.340	
110	681.930	681.65	0.280	
120	682.015	681.80	0.215	
130	682.115	681.95	0.165	
140	682.240	682.10	0.140	
150	682.345	682.25	0.095	
160	682.400	682.40	0.000	
170	682.520	682.55		0.03
180	682.640	682.70		0.06
190	682.730	682.85		0.12
200	682.825	683.00		0.175

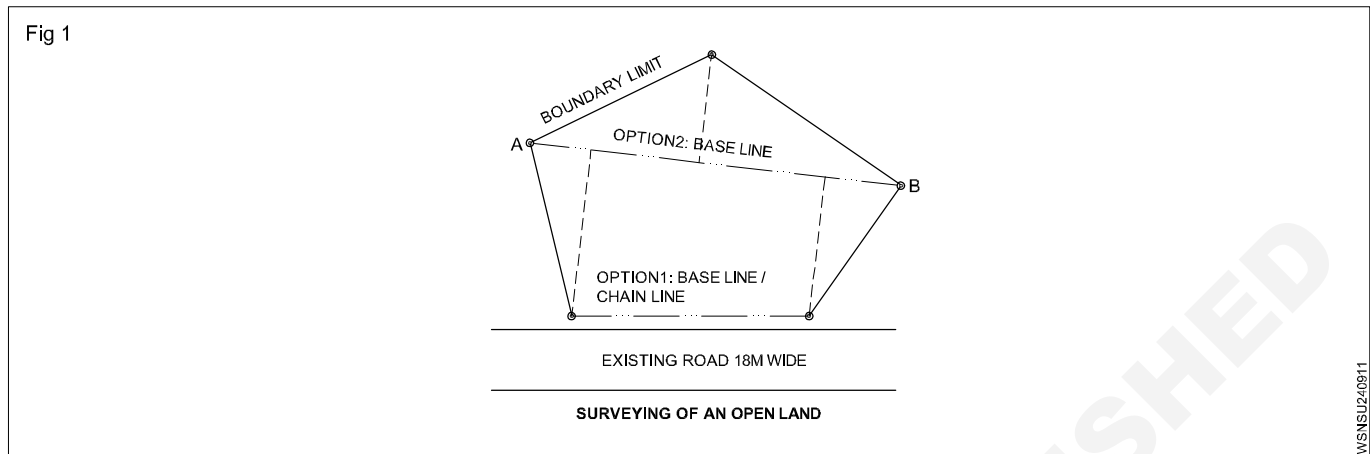
Inspect the site and take measurement = 2 persons (Surveyor + Helper) - 2 days

Preparation of tabulate column and calculation = 2 days

Leveling the ground = 5 persons x 2 days

**Estimation and Costing - Simple estimation of the requirement of material etc., as applicable to the trade - Chain survey to an open land for layout plots**

Chain survey to an open land for layout plots. (Fig 1)



1	Fix a baseline is boundary line (or) diagonal to the boundary corners (Fig)	2 hours
2	Same baseline/chain line to be retained (or) permanently marked for laying of layout plots	1 hours
3	Locate all boundary corners and existing (or) access roads, natural and man made features if it required.	1 hour
4	Prepared a detailed survey drawing of the land	4 hours
5	Prepare the layout plan as per requirements with norms which is applicable to the particular land.	4 hours
6	Planning to be made without wasting of any land necessary accesses roads and regular plots.	2 hours
7	To mark the lauyout at the ground take right angle offsets and with reference to bease line/chain line	2 hours
<b>Total hours</b>		<b>16 hours</b>

**Calculation**

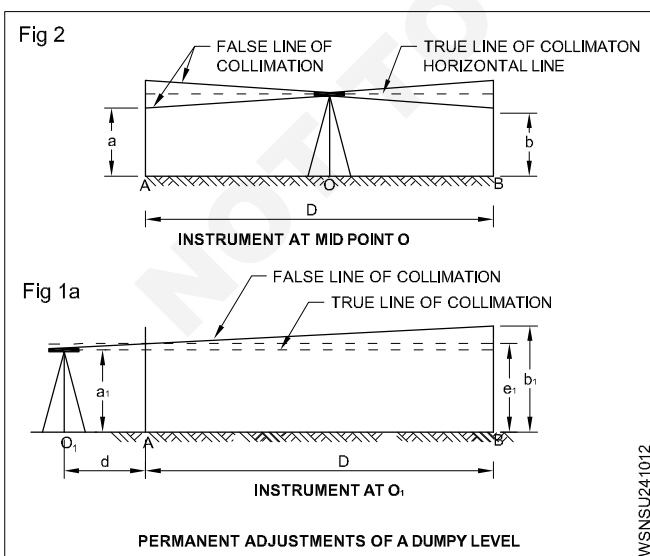
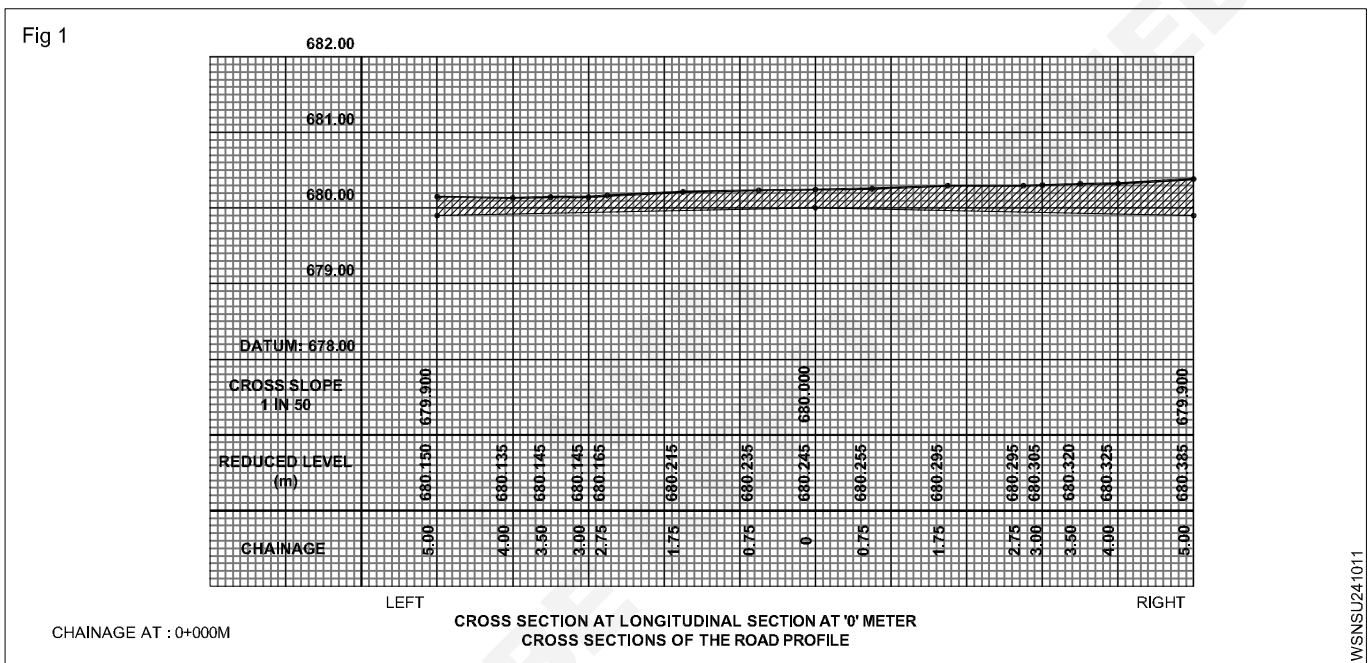
Inspect the site and take measurement	= 2 persons
	= Surveyor + Helper
Surveyor charge	= Rs.2000
Helper charge	= Rs.1000
Prepare the layout plan drawing	= Rs.5000
<b>Total charge</b>	<b>= <u>Rs.8000</u></b>



Estimation and Costing - Problems on estimation and costing - Practice profile levelling or longitudinal and cross section levelling - plotting profile

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200	682.825	683.00		0.175

### Calculation

Inspect the site and take measurement = 2 persons (Surveyor + Helper) - 2 days  
 = Rs.2000 + Rs.1000  
 For 2 days = Rs.2000 x 2 + Rs.1000 x 2  
 = Rs.4000 + Rs.2000  
 = Rs.6000  
 Preparation of tabulate column and calculation = 2 days  
 = Rs.1000 x 2 = Rs.2000  
 Leveling the ground = 5 persons x 2 days  
 Labour charge = Rs.5000 x 2  
 = Rs.10,000  
 Total = Rs.6000 + Rs.2000 + Rs.10,000  
 = **Rs.18,000**

Estimation and Costing - Problems on estimation and costing - Prepare a site by the help of mouza map

Prepare a site by the help of mouza map. (Fig 1)

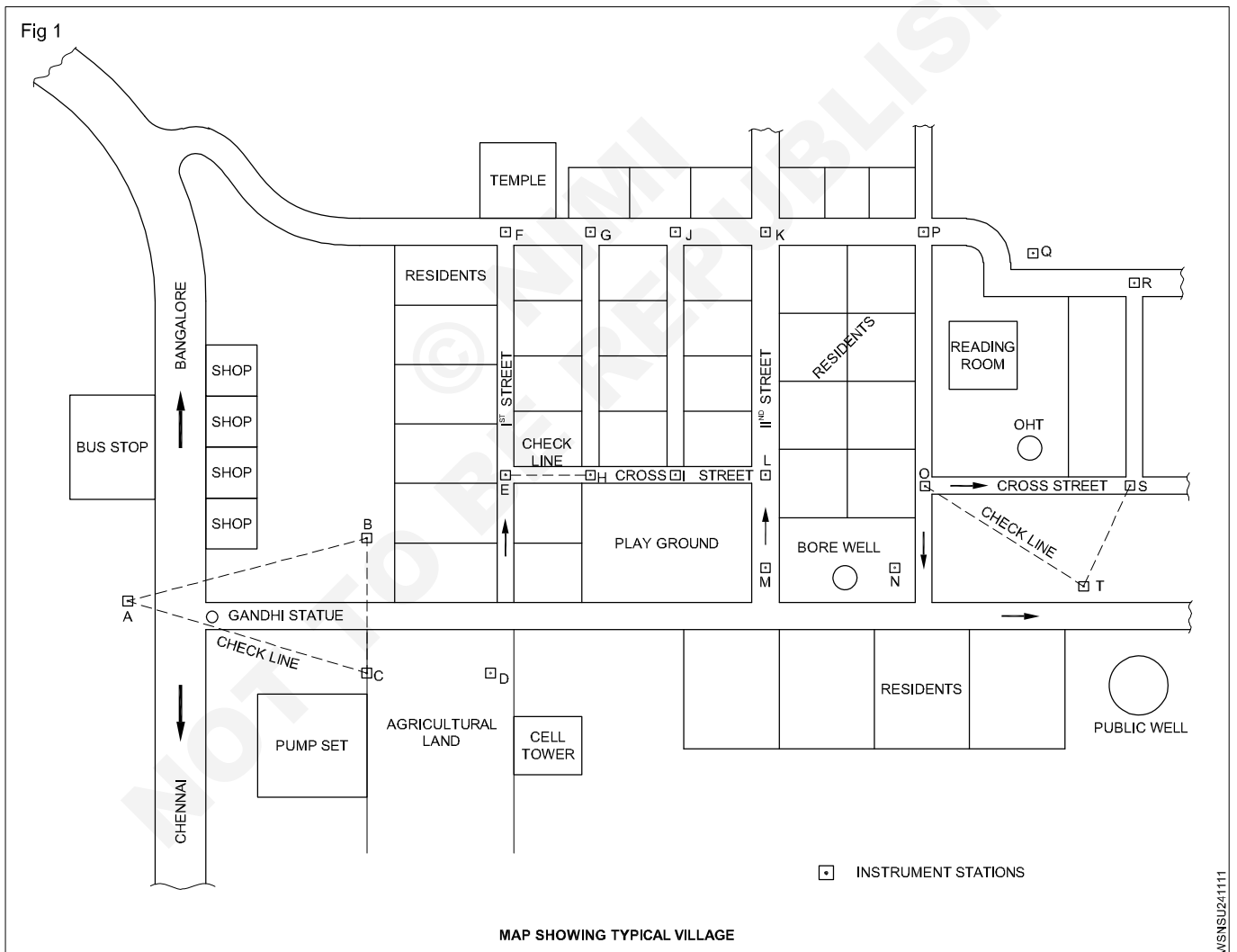
Tools/Instruments - Surveyor Owns

- Plane table with tripod
- Alidade
- Spirit level
- Trough compass
- Plumbing fork with plum bob
- Measuring (30m) steel tape
- Pegs

- Ranging rods
- Computer with CAD software

Materials

- Drawing sheet A2
- Set of scale
- Pencil, Eraser etc.,
- Cello tape



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### Prepare a map of a village (Open traverse)

- 1 Select the instrument stations A,B,C etc., after inspecting the site to get rough idea about the village. (Figure showing a typical village)
- 2 Set up the table 'A'.
- 3 Centre and level the table and mark point 'a' suitably on the sheets.
- 4 Mark the direction of the magnetic north on the sheet.
- 5 Place a ranging rod on station 'B'.
- 6 Keep the alidade on 'a' sight the ranging rod at 'B'.
- 7 Draw a ray along the fiducial edge of alidade.
- 8 Measure the distance AB with tape.
- 9 Select suitable scale and draw distance 'ab' on sheet.
- 10 Locate the surrounding details by radiation or intersection.
- 11 Shift the instrument and set it up at 'B'
- 12 Centre and level the table.
- 13 Orient the table by back sighting on 'A'
- 14 Clamp the table.
- 15 Place the ranging rod on station 'C'.
- 16 Keep the alidade pivoted on 'b' sight the station 'C'.
- 17 Draw a ray along the fiducial edge of the alidade.
- 18 Measure the distance 'BC' with tape.
- 19 Set off distance 'bc' on the ray.
- 20 Locate the surrounding details by radiation or intersection.
- 21 Proceed the same process until all the remaining stations and surrounding details are to be plotted.
- 22 Prepare the map drawing using CAD software.
- 23 At any station the work can be checked by taking sights to two or more preceding stations visible from the station occupied
- 24 When no station is visible then any well defined points or objects such as corner of building electric posts etc., may be used for checking.
- 25 Radiation is applicable when the distance between the object and instrument station can be measured easily.
- 26 Intersection is applicable when the object is far away from the instrument station which are not possible to measure directly.

### Calculation

Inspect the site and measure the area - 2 persons	=	Surveyor + 2 Helper - 1 day
		Sureveyor = Rs.2000/day
		Helper = Rs.1000/day
Prepare the map drawing using CAD software	=	Rs.2000
		Total = Rs.2000 + Rs.2000 + Rs.2000
		= <b>Rs.6000</b>

**Estimation and Costing - Problems on estimation and costing - Prepare a detailed road project (more than 1 km)**

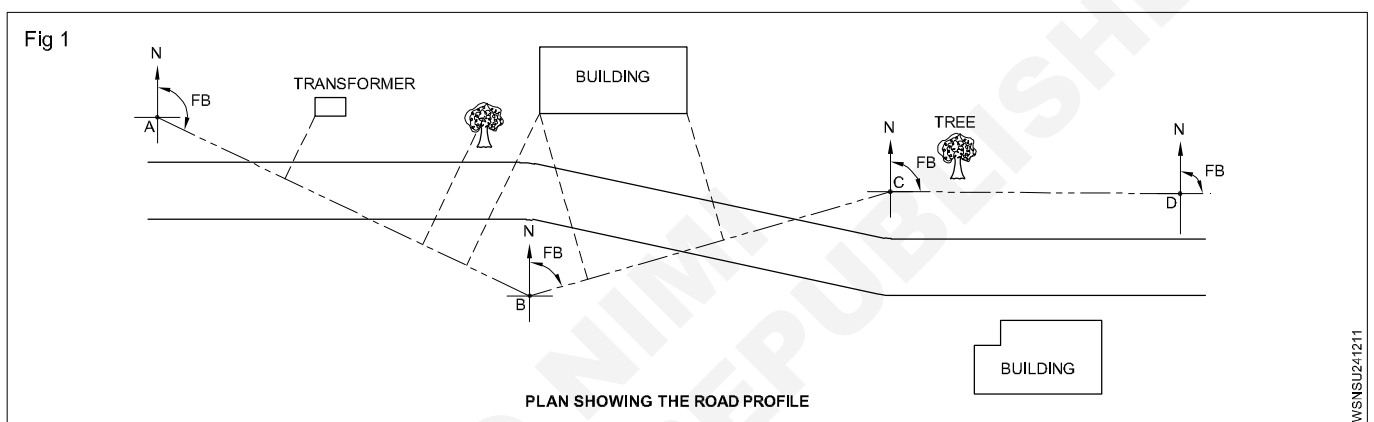
Prepare a detailed road project (more than 1 km). (Fig 1)

**Tools/Instruments - Surveyor Owns**

- Prismatic compass - 1 No.
- Ranging rods - as reqd.
- Wooden peg - as reqd.
- Chain & tape - 30m with arrows - 10 arrows
- Cross - staff - 1 No.

**Materials**

- Drawing sheet A3 - 1 No.
- Field book - 1 No.
- Pencil HB - 1 No.
- Eraser - 1 No.
- Cello taps - 1 No.
- Set of scale - 1 Set



**Prepare a road map with details along the road**

- 1 Make a visit to the existing road for the purpose of identifying the objects and special features located on the sides of road to be surveyed.
- 2 Select an instrument station 'A' at the beginning of road which gives maximum details for mapping.
- 3 Select the other stations B,C,D etc.,
- 4 Set up the instrument over the selected station 'A'
- 5 Fix a ranging rod at station 'B'
- 6 Sight station 'B' and take the bearing note it in the field book.
- 7 Run the chain along 'AB' and take off-sets of the objects on both sides of the road.
- 8 Observations should be recorded in the filed book.
- 9 Shift the instrument to station 'B'.
- 10 Take the bearing by back sighting 'A'.
- 11 Check it with the fore bearings of 'AB'

- 12 Fix a ranging rod at 'C'
- 13 Sight station 'C' and take the bearing of 'BC' and enter it.
- 14 Run the chain along BC and take offsets of objects on both sides of the road.
- 15 Similarly the station points as fixed more than 1 km length.
- 16 All the bearings should be entered in the field book as per the table given below.
- 17 The offsets taken for different objects are entered as a chain survey field book.
- 18 Plot the road map based on the bearings and offsets.

### Calculation

Inspect the site and take measurement = 1 Surveyor + 2 Helper - 2 day

Surveyor = Rs.2000 x 1

= Rs.2000

For 2 days = Rs.2000 x 2

= Rs.4000

Helper = Rs.1000 x 2

= Rs.2000

For 2 days = Rs.2000 x 2

= Rs.4000

Prepare the drawing = 2 days = Rs.2000

Total = Rs.4000 + Rs.4000 + Rs.2000

= **Rs.10,000**

Estimation and Costing - Problems on estimation and costing - Transmission line site survey - Foundation pit point

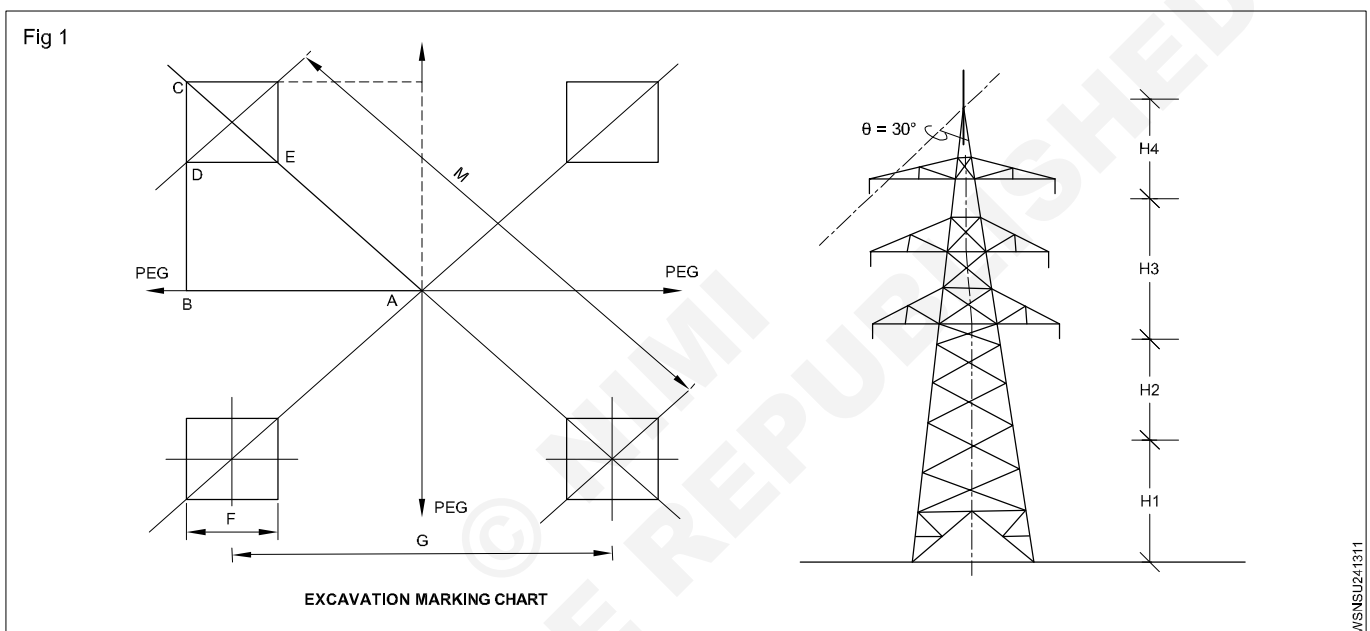
Transmission line site survey - Foundation pit point. (Fig 1)

Tools/Instruments - Surveyor Owns

- Drawing board with drafting machine - 1 No.
- Metric scale 30 cm - 1 Set
- Sets square (45° & 60°) - 1 Set
- Instrument box - 1 No.

Materials

- A2 size drawing paper - 2 Nos.
- Eraser - 1 No.
- Pencils H & HB - 1 No.each
- Cello tape - as reqd.



Mark tower foundation pit points

- 1 Foundation of the location is carried out. This is based on the specific foundation drawing of the type and make of the tower which is to be erected at that location. The tower legs, footings and faces are designated.
- 2 Designation of Tower Legs, Footing and Face working space. No margin is necessary in case of undercut foundations.
- 3 The excavation pit marking drawing indicates the distance of centres, sides and corners of the pits with reference to the centre point of the tower. These distances are measured and each pit boundary is marked in the field by means of spade or pick axe along the sides of the pit.
- 4 The pit marking shall be carried out according to the pit marking drawing which is prepared from the foundation drawing of the tower. The size of the pit, in case of open cut foundations, is worked out by adding 150 mm to the sides of the base pad on all the four sides for allowing
- 5 A typical excavation pit marking drawing is given
- 6 Designation of Tower Legs, Footing and Face
- 7 Excavation marking chart

- 8 From the dimensions shown in the drawing, the triangle ABC is first marked with the help of a measuring tape. The distance CD', equal to F, is then marked. The sides DE and D'E, both equal to F, are then marked. The procedure is repeated for marking the other three pits. 1.7 The dimension G shown in the drawing is the centre to centre distance between stubs of the tower at their lowest point. The dimension M is the diagonal distance between the ends of the stubs of the tower. The excavation pit marking drawing is prepared on the basis of these dimensions.
- 9 The depth of actual excavation at the pit centre is measured with reference to the ground level at the centre of the tower location. A typical example of determining the actual depth of excavation of the pits is shown below.

### Calculation

1	Locatin finalize	= 1 day - 2 persons
2	Marking	= 1 day - 2 persons
3	Excuvation 4 bits	= 2 days - 8 persons
4	Concrete	= 1 day - 6 persons
	Total persons	= 18 persons
5	Labour charge	= Rs.1000/persons
		= 18 x Rs.1000
		= Rs.18,000
6	Materials, Irons rod, Sand, Jally	= Rs.20,000
	Total cost	= Material cost + labour charge
		= Rs.20,000 + Rs.18,000
		= <b>Rs.38,000</b>



## Estimation and Costing - Problems on estimation and costing - Calculate the discharge of a river

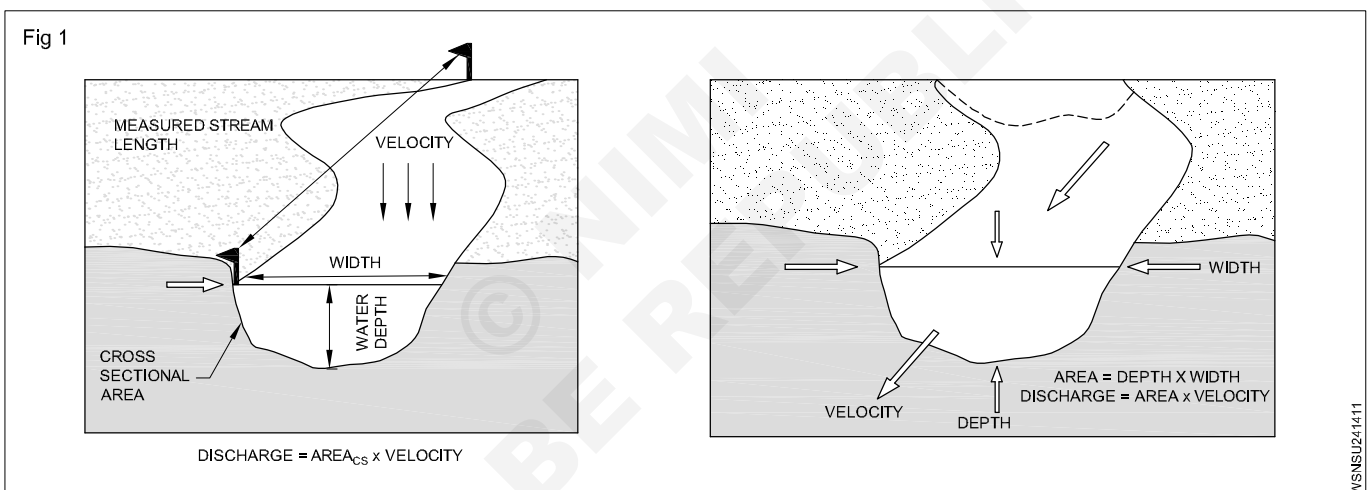
Calculate the discharge of a river. (Fig 1)

### Tools/Instruments - Surveyor Owns

- Drawing board with drafting machine - 1 No.
- 30 cm scale - 1 Set
- Sets square (45° & 60°) - 1 Set
- Instrument box - 1 No.
- 2 flags to mark the start and end of the stream
- Monitoring section
- Measuring tape
- Orange or other float device

### Materials

- A2 size drawing paper - 1 No.
- Eraser - 1 No.
- Pencils H & HB - 1 No. each
- Cellotape - as reqd.
- Appropriate footwear (boots or water shoes)
- Personal flotation device (when requested by teacher)
- Stopwatch
- Calculator



### Set-up

- 1 Locate a straight section of stream atleast, 20 feet long. Mark the start and end of the length of stream with flags.
- 2 Measure the width of the stream and record data.
- 3 Measure the depth (in tenths of an inch) of the stream at 1 foot intervals. Record data.
- 4 Observe the bottom of the stream. Define below the best description of the stream bottom.
  - a rough, loose rocks, coarse gravel, weeds.
  - b Smooth mud, sand, bedrock.

### Measuring velocity

- 5 Release your float 3-4 feet upstream of the "Start" flag.
- 6 Measure the time taken by the float to travel the distance between flags. With the helpers in small groups of 4 to 6 persons, have one person in the water ready to release the float object, one on shore watching the starting line, another watching the finish line, one using the stopwatch and all others in the water behind the finish line to catch the object. Time the distance between flags.

- 7 Repeat step 4 and 5 at three additional locations across the stream,
- 8 Multiply the average depth of the stream by the width of the stream to find the area in ft<sup>2</sup>.
- 9 Divide the distance travelled by float with the average travel time to find the velocity of stream in ft/sec.
- 10 Multiply the velocity of the stream by a correction factor. This is corrected velocity of the stream.
  - a Choose a correction factor of 0.8 for stream bottoms with rocks, coarse gravel or weeds.
  - b Choose a correction factor of 0.9 for stream bottoms with smooth mud, sand or bedrock.
- 11 Multiply the corrected velocity of the stream by the area of the stream to obtain ft<sup>3</sup>/sec. This is the measure of stream flow in cubic feet per second!
- 12 Based on your data analysis, write a conclusion for this activity.

### Calculation

Engineer	= 1 Person	= 1 x Rs.3000
		= Rs.3000
Surveyor	= 1 Person	= 1 x Rs.2000
		= Rs.2000
Helper	= 6 Persons	= 6 x Rs.1000
		= Rs.6000
Total charge	= Rs.3000 + Rs.2000 + Rs.6000	
		= Rs.11,000
For 2 days	= 2 x Rs.11,000	
		= <b>Rs.22,000</b>